Macroprudential Policy Under Diagnostic Expectations*

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Abstract

In this paper, we incorporate diagnostic expectations into the standard small open economy model with an occasionally binding constraint. Diagnostic expectations are a forward-looking model of belief formation characterized by an overreaction to recent news. We find that diagnostic expectations exacerbate crisis dynamics and generate realistic boom-bust credit cycles through the interaction of overreaction to news, neglect of risk, and the borrowing constraint. Optimal macroprudential policy mainly addresses the pecuniary externality and corresponding overborrowing in normal times, while mitigating underborrowing induced by over-pessimism during financial crises. Policymakers often advocate for a macroprudential policy of "leaning against the wind," while the standard quantitative model typically recommends "fueling the economic boom." By introducing extrapolative belief distortions, our framework reconciles this disconnect and aligns the quantitative policy prescription with conventional policymaking wisdom.

JEL Classification: G18, G15, G41, F44

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1 Introduction

Two paradigms dominate the study of financial crises. The first, formalized in the "sudden stops" literature, focuses on a fire-sale amplification mechanism, as described by Fisher (1933), where individually rational deleveraging decisions collectively depress asset prices and deepen a downturn. The corresponding policy prescription is to address the pecuniary externality which arises because individuals fail to internalize the effect of their decisions on equilibrium prices (Bianchi, 2011, Bianchi and Mendoza, 2018). The second paradigm, following Minsky (1977) and Kindleberger (1978), attributes credit cycles to waves of excessive optimism and pessimism and describes how a euphoric, credit-fueled boom can suddenly reverse into a panic-driven crash. This second view suggests that policy should directly counter these behavioral distortions (Farhi and Werning, 2020). While these two frameworks are often treated as distinct, a crucial gap exists in formally linking them.

Identifying the specific behavioral friction that gives rise to Minskyan boom-bust dynamics within a quantitative model remains a subject of intense research. Diagnostic expectations—a psychologically-grounded theory in which agents systematically overreact to new information (Tversky and Kahneman, 1983, Bordalo et al., 2018)—offer a compelling and tractable foundation. As shown by Bordalo et al. (2022), models based on diagnostic expectations feature forward-looking agents and explicitly address the Lucas (1976) critique, distinguishing them from alternative approaches such as adaptive expectations. Furthermore, diagnostic expectations closely align with empirical evidence, as they successfully account for the overreactions documented in survey, experimental, and news announcement data (Bordalo et al., 2020, Afrouzi et al., 2023, Kwon and Tang, 2025).

To bridge the gap between these two paradigms, we embed diagnostic expectations (henceforth, DE) within a standard small open economy model of Bianchi (2011) that features an occasionally binding collateral constraint. Our approach creates a unified framework where belief distortions drive the initial boom and subsequent bust, while a pecuniary externality dramatically amplifies the crisis. This framework allows us to dissect the interaction between these two core mechanisms and provide new insights into whether macroprudential policy should aim to tame expectations, mitigate fire sales, or both. Additionally, while standard models exhibit credit busts during crises, they often struggle to generate the credit booms that precede them. In contrast, our model with a reasonably calibrated diagnosticity parameter reproduces realistic boom-bust credit cycles. It successfully generates significant pre-crisis debt accumulation driven by over-optimism, followed by sharp credit busts during the crisis.

Moreover, our model with DE encompasses endogenous states of both overborrowing and underborrowing, given a standard parameterization. Borrowing decision rules are determined endogenously as an outcome of the recursive equilibrium with diagnostically distorted beliefs. As a result, diagnostic agents tend to borrow more than optimal when they are optimistic (in response to good news) and less when they are pessimistic (in response to bad news). On average, agents exhibit overborrowing behaviors over business cycles and underborrowing behaviors during crises. This result complements Schmitt-Grohé and Uribe (2021), who find that the standard small open economy model with a flow constraint can generate multiple equilibria, including overborrowing and underborrowing, under specific parameterizations.

Additionally, incorporating DE into the standard model designed to study macroprudential policy helps reconcile the disparity between theory and practice. Schmitt-Grohé and Uribe (2017) show that the model of Bianchi (2011) recommends tightening the macroprudential regulation during crises, which is at odds with the conventional wisdom of policymakers. We bridge this gap by embedding DE into the model of Bianchi (2011). In the baseline exercise, we focus on the case where the social planner holds rational expectations (henceforth, RE) while individual households have DE. We analytically show that optimal debt taxes implemented in the decentralized competitive equilibrium address two components: (i) the pecuniary externality adjusted by interactions with DE, and (ii) pure expectation corrections between the rational social planner and diagnostic households. In equilibrium, the optimal debt tax rate rises when the amount of debt increases or when households become optimistic in a good news state. Conversely, the debt tax rate should be reduced when the amount of debt decreases or when households become pessimistic. Interestingly, the debt tax rate can even become negative (implying subsidies) when households are pessimistic in a bad news state, indicating that this state is in the underborrowing region.

As a result, the optimal debt tax becomes countercyclical with regard to output (consistent with the conventional wisdom of leaning against the wind) in the model with DE, while the Bianchi (2011) model generates a procyclical optimal debt tax. This is because, under DE, debt tax rates need to be raised in response to good news and associated overborrowing, which often coincides with output booms. In contrast, the Bianchi (2011) model suggests increasing the debt tax to address intensified overborrowing that typically leads to crises, a period when recessions are likely and the borrowing constraint is also likely to bind.

To reconcile the cyclicality of optimal policy with conventional wisdom, Schmitt-Grohé and Uribe (2017) modifies the Bianchi (2011) model by adding interest rate shocks and altering households' discount factors. However, these modifications do not change the sign of the cyclicality of optimal policy and often fail to match data, such as underpredicting the frequency of financial crises. In contrast, our model maintains its explanatory power in matching business cycles and crisis dynamics observed in data while successfully reproducing the countercyclical optimal tax, aligning with conventional wisdom.[1] Our results on the countercyclicality of optimal debt taxes are robust to alternative parameterizations, including different levels of diagnosticity, tightness of borrowing constraints, discount factors, and the persistence and volatility of income shocks. We also consider an extension of the model that allows for production in both tradable and non-tradable sectors. We find that the qualitative implications for optimal tax cyclicality remain broadly similar, especially for higher values of the diagnosticity parameter.

Using simulated data from our model, we quantify the role of DE in macroprudential policy over business cycles and during financial crises. During regular business cycles, the optimal policy primarily addresses overborrowing driven by the pecuniary externality. During crises, however, the policy's focus shifts to addressing the sharp reversal in sentiment, as the expectation correction term begins to dominate the pecuniary externality correction term. Moreover, we find that the interaction between the pecuniary externality and DE is muted by a negative correlation between the overborrowing incentives from these two sources. Specifically, following a negative income news, agents are driven to borrow more for consumption smoothing (strengthening the pecuniary externality), but they also borrow less due to over-pessimism.

Furthermore, we show that debt taxes become substantially acyclical around crises when the paternalistic policymaker (a rational social planner with diagnostic households) can only implement a simple Taylor-type debt tax rule. Ideally, the paternalistic planner corrects borrowing incentives and inefficiencies arising from expectation disparities, meaning the optimal tax rule should depend on the current news state (either bad or good). However, the simple rule implemented by the policymaker might not be dependent on the current news, as is often the case in the real world. This suggests that it is best not to change debt tax rates significantly when fully state-contingent tax instruments are not implementable, since imposing taxes incorrectly across states leads to significant welfare loss. This result aligns with empirical evidence showing that the measure of capital controls is almost acyclical around crises in most countries (Fernández et al., 2015, Acosta et al., 2023).

Finally, we employ a global solution method to analyze the model with DE, focusing on the case where agents form their expectations about an AR(1) exogenous process. In this environment, we first show that the sequential problem can be recast as a recursive problem under under the assumption that agents' memory is limited to the immediate past. We then illustrate how this approach can be used to solve the model with DE and discuss how it differs from the solution method based on ARMA(1,1) misperceptions.

[1] Flemming et al. (2019) also obtain countercyclical optimal debt tax by including a permanent component into the endowment process. Conversely, our model achieves countercyclicality of the optimal tax without additional exogenous shocks, as it incorporates endogenous shifts in borrowing decision rules driven by an extrapolative expectation mechanism.

Related Literature This paper relates to two branches of literature: (i) optimal macroprudential policy and (ii) financial crises with deviation from RE.

First, we contribute to the literature on optimal macroprudential policy. One branch rationalizes interventions in the competitive equilibrium based on pecuniary externalities (Lorenzoni, 2008, Bianchi and Mendoza, 2018, Dávila and Korinek, 2018, Ottonello et al., 2022). Numerous papers build on the canonical model of pecuniary externality by Bianchi (2011) to explore the effects of various frictions and environments on crisis dynamics and optimal policy.[2] Bianchi et al. (2016) find that incorporating news shocks with a signal extraction problem into the model can strengthen incentives to borrow during good times, increasing vulnerability to crises.[3] Schmitt-Grohé and Uribe (2017) show that the model of Bianchi (2011) generates optimal debt taxes that are negatively correlated with output (procyclical), which is at odds with the conventional wisdom of policymakers who advocate for countercyclical policies. Flemming et al. (2019) show that incorporating a permanent component into the endowment process recovers the conventional wisdom of optimal macroprudential policy. Kwak (2020) also finds that firm's endogenous default risks could reproduce countercyclical optimal debt taxes. Our contribution is to identify an ingredient, DE, which reconciles the existing theoretical framework with the practice of policymakers.

We do not assume any information frictions, which means that agents can observe the current level of income correctly. In contrast, others study the role of information frictions or learning in open economy business cycles (Boz, 2009, Boz et al., 2011, Bianchi et al., 2012, Akıncı and Chahrour, 2018). Boz (2009) analyzes the role of optimism in emerging market crises by incorporating information frictions with Bayesian learning. She shows that when the economy becomes over-optimistic due to a sequence of positive signals, a small negative noise shock can lead to financial crises. Boz et al. (2011) and Herreño and Rondón-Moreno (2025) investigate the role of pecuniary externality and information frictions in shaping optimal macroprudential policy.

Second, we contribute to the literature investigating financial crises or business cycles in the presence of behavioral biases (Bordalo et al., 2018, 2021, Gennaioli et al., 2015, Maxted, 2024, L'Huillier et al., 2024, Bianchi et al., 2024a, Camous and Van der Ghote, 2022). We examine the normative implications arising from the interaction between pecuniary externalities and behavioral biases. A key contribution of our work is the explicit derivation of the optimal debt tax, which we decompose into two components: a pecuniary externality term and an ex-

[2] Recent research in these areas often employs global solution methods to analyze financial crises, capturing the tail risk and the nonlinear dynamics of such events (Bianchi, 2011, Bianchi and Mendoza, 2018, Flemming et al., 2019, Herreño and Rondón-Moreno, 2025, Arce et al., 2025).

[3] See Bianchi et al. (2012), Seoane and Yurdagul (2019), and Rojas and Saffie (2022) among others for additional modifications to models featuring optimal macroprudential policy.

pectation correction term. The closest related work is Fontanier (2025), who studies optimal macroprudential policy under a general class of deviations from RE, but focuses primarily on the implications of correcting behavioral externalities. Furthermore, most recent applications of DE have focused on closed-economy settings, leaving their implications in small open economy models largely unexplored. An exception is Na and Yoo (2025), who incorporate DE into a standard small open economy framework and solve it using local method. They show that this modification improves the model's ability to match key business cycle moments without resorting to trend-driven TFP shocks. We contribute to the literature by incorporating DE into a small open economy model with an occasionally binding constraint and solving it using the global method.[4] Using the global method is essential to understand endogenous rare events, as pointed out in de Groot et al. (2025).

The remainder of the paper is organized as follows. Section 2 introduces the concept of DE and presents an illustrative 3-period model. Section 3 develops the baseline small open economy model with DE, defines the equilibrium with diagnostic agents, and derives the optimality conditions for both private agents and the constrained social planner. Section 4 characterizes optimal macroprudential policy of rational and diagnostic social planners and highlights the implications of correcting belief distortions. Section 5 presents the main quantitative analysis. Section 6 concludes. The Appendix provides additional details on the quantitative analysis and supplementary results, including the nonlinear effects of DE.

2 Diagnostic Expectations and Overborrowing

2.1 Diagnostic Expectations

Diagnostic expectations build on the *representativeness heuristic* of Tversky and Kahneman (1983). The central idea is that individuals place disproportionate weight on new information that appear highly representative of recent events while neglecting less salient information. In this sense, DE capture a systematic overreaction to recent news relative to past information. Bordalo et al. (2018) formalize this idea into an economic model of belief formation, and subsequent work demonstrates its ability to generate asset pricing anomalies, boom-bust credit cycles, and business cycle fluctuations (Bordalo et al., 2019, 2024, 2021, 2020).

To see how DE work in belief formation regarding aggregate economic conditions, consider a random variable x_t that follows the AR(1) process:

$$x_t = \rho x_{t-1} + \varepsilon_t, \tag{1}$$

[4] Niemann and Prein (2024) also solve the sovereign default model under DE using global method.

where $\rho \in (0,1)$ and ε_t is Gaussian with a mean zero and a standard deviation σ. Following Bordalo et al. (2018), we define the diagnostically distorted distribution of x_{t+1} as:

$$f_t^\theta(x_{t+1}) = f(x_{t+1}|G_t) \cdot \left[\frac{f(x_{t+1}|G_t)}{f(x_{t+1}|-G_t)}\right]^\theta \cdot \frac{1}{Z}, \tag{2}$$

where the normalizing constant Z ensures that $f_t^\theta(x_{t+1})$ integrates to one, and G_t and $-G_t$ represent the current and reference state respectively. The crucial parameter θ captures the severity of evaluating by representativeness. When $\theta = 0$, the agent has no belief distortion and thus forms RE given the current state $G_t \equiv \{x_t = \hat{x}_t\}$ ($\hat{x}_t$ is the realized current state). When $\theta > 0$, the agent's belief is distorted by a representative state. From now on, we assume that the reference state is given by $-G_t \equiv \{x_t = \rho\hat{x}_{t-1}\}$, which reflects information at time $t-1$ with no news.[5] Then, a diagnostic agent perceives the distribution of x_{t+1} by overweighting the likelihood of representative states while underweighting the likelihood of non-representative states. This also implies that the current news not only modifies the objective probability of specific states but also influences the degree of attention the agent allocates to these states. In this regard, the positive news induces a diagnostic agent to neglect left-tail risks such as financial crisis events.

From equation (2), we can express the DE of x_{t+1}, denoted by $E_t^\theta[x_{t+1}]$, using the formula of the RE, $E_t[x_{t+1}]$:

$$E_t^\theta[x_{t+1}] = E_t[x_{t+1}] + \theta(E_t[x_{t+1}] - E_{t-1}[x_{t+1}]). \tag{3}$$

This equation shows how agents' expectations are distorted relative to the RE benchmark. From the equation (1), we can rewrite the DE of x_{t+1} as:

$$E_t^\theta[x_{t+1}] = \rho x_t + \theta(\rho x_t - \rho^2 x_{t-1}) = \rho x_t + \theta\rho\varepsilon_t. \tag{4}$$

See Bordalo et al. (2018) for the proof.[6]

As long as the persistence parameter is positive ($\rho > 0$), the agent with DE becomes over-optimistic and neglects left-tail risks in response to good news ($\varepsilon_t > 0$). When the agent receives bad news ($\varepsilon_t < 0$), she becomes overly pessimistic.

[5] Bianchi et al. (2024a) generalize the reference state to time $t-J$ where $J > 1$ ("distant memory") and show that it can generate hump-shaped dynamics in New Keynesian models. For computational tractability when using the global method, we focus on the case of immediate memory ($J = 1$). As shown in Appendix B.1, our model can be cast into the recursive formulation under the immediate memory, which facilitates the use of the global method under DE.

[6] The variance of $f_t^\theta(x_{t+1})$ does not depend on σ^2 under equations (1) and (2). Bordalo et al. (2018) discuss time-varying variance by assuming a GARCH process in the appendix, while Bianchi et al. (2024b) extend the DE framework to incorporate time-varying variance and show that when uncertainty is high, overreaction becomes more pronounced.

2.2 Diagnostic Expectations and Overborrowing: A 3-period Model

For illustration, we present a three-period model in which agents receive endowments in each period. They allocate endowments into consumption and savings. We build on the simple model of Bianchi et al. (2024a) by adding the occasionally binding borrowing constraint and persistence to the endowment process. The purpose of this illustration is to show how DE generate overborrowing in equilibrium and amplify crisis probability by interacting with the occasionally binding constraint. We will incorporate the pecuniary externality due to price variation in the full quantitative model.

The endowment process y_t follows an AR(1) process for each period $t = 1, 2, 3$:

$$y_t = \rho y_{t-1} + \varepsilon_t, \quad \text{where } 0 \leq \rho < 1 \text{ and } \varepsilon_t \sim N(0, \sigma^2). \tag{5}$$

Following Bianchi et al. (2024a), we assume quadratic utility:

$$u(c) = \delta c - \frac{\gamma}{2} c^2,$$

where c is consumption; δ and γ are constant and positive.

The agent under RE solves the following utility maximization problem:

$$\begin{aligned} &\max_{b_1, b_2} \{u(c_1) + E_1 [u(c_2) + u(c_3)]\} \qquad (6) \\ &\text{s.t. } c_1 = y_1 + b_0 - b_1, \\ &\quad c_2 = y_2 + b_1 - b_2, \\ &\quad c_3 = y_3 + b_2 - b_3, \\ &\quad b_2 \geq -\kappa y_2. \end{aligned}$$

For simplicity, we assume the discount factor (β) is unity and the net interest rate (r) is zero. A fraction κ of second-period income, y_2, can be collateralized to issue debt (thereby reducing the bond holding, b_2).[7]

In the following section, we compare solutions to savings (b) under (i) RE, (ii) DE on an exogenous variable (endowment, y), and (iii) DE on an endogenous variable (consumption, c). Our quantitative model assumes DE on an exogenous variable like Bordalo et al. (2021), while L'Huillier et al. (2024) and Bianchi et al. (2024a) assume DE on endogenous variables. In the

[7] Since the collateral constraint depends only on a current variable, lenders do not need to form expectations to determine its tightness. Expectations would matter if the constraint incorporated future variables, such as borrower income or collateral value. For instance, lenders may require that future income be sufficient to repay debt, as in Ottonello et al. (2022). In such cases, the DE operator reflecting the lenders' expectations can be applied to the collateral constraint.

following propositions and proofs, we show that solutions rarely change qualitatively under DE on either exogenous variables or endogenous variables, as long as income persistence (ρ) is larger than zero.

We have the optimal borrowing decision rules of the RE agents and associated probability of crises in the following proposition.

Proposition 1. *The equilibrium under RE is characterized as follows.*

Non-binding borrowing constraint *If the borrowing constraint in period 2 does not bind, the solution is given by*

$$b_1^* = \frac{2}{3}b_0 + \frac{2}{3}\left(1 - \frac{\rho(1+\rho)}{2}\right)y_1, \qquad b_2^* = \frac{1}{2}b_1 + \frac{1}{2}(1-\rho)y_2. \tag{7}$$

Binding borrowing constraint *If the borrowing constraint binds in period 2, the solution is*

$$b_1^* = \frac{1}{2}b_0 + \frac{1}{2}\left[1 - (1+\kappa)\rho\right]y_1, \qquad b_2^* = -\kappa y_2, \tag{8}$$

where the borrowing constraint binds if and only if

$$\varepsilon_2 < \bar{\varepsilon}_2^{RE} \equiv -\frac{b_1 + (1-\rho+2\kappa)\rho y_1}{1-\rho+2\kappa}. \tag{9}$$

Crisis Probability *The probability that the borrowing constraint binds (the probability of crises) is given by*

$$\Pr(\varepsilon_2 < \bar{\varepsilon}_2^{RE}) = F(\bar{\varepsilon}_2^{RE}), \tag{10}$$

where $F(\cdot)$ denotes the cumulative density function of news at time 2, ε_2.

Moreover, if the constrained social planner takes the borrowing constraint as given, the competitive equilibrium coincides with the constrained-efficient allocation.

Proof. See Appendix A.1. ■

We now assume that DE are applied to exogenous variables, which aligns with the approach of Bordalo et al. (2021). The DE agent solves the following problem with the modified expectation operator:

$$\max_{b_1^\theta, b_2^\theta} \left\{ u(c_1^\theta) + E_1^\theta \left[u(c_2^\theta) + u(c_3^\theta) \right] \right\} \tag{11}$$

$$\begin{aligned} \text{s.t. } c_1^\theta &= y_1 + b_0^\theta - b_1^\theta, \\ c_2^\theta &= y_2 + b_1^\theta - b_2^\theta, \\ c_3^\theta &= y_3 + b_2^\theta - b_3^\theta, \\ b_2^\theta &\geq -\kappa y_2, \end{aligned}$$

where agents form DE on endowment process as follows:

$$E_t^\theta(y_{t+1}) = E_t(y_{t+1}) + \theta[E_t(y_{t+1}) - E_{t-1}(y_{t+1})], \quad \theta \geq 0. \tag{12}$$

Proposition 2. *The equilibrium under DE on exogenous variables is characterized as follows.*

Non-binding borrowing constraint *If the borrowing constraint in period 2 does not bind, the solution is*

$$b_1^{\theta *} = \frac{2}{3} b_0 + \frac{2}{3}\left(1 - \frac{\rho(1+\rho)}{2}\right) y_1 - \frac{1}{3}\rho(1+\rho)\theta\varepsilon_1, \tag{13}$$

$$b_2^{\theta *} = \frac{1}{2} b_1 + \frac{1}{2}(1-\rho) y_2 - \frac{1}{2}\rho\theta\varepsilon_2. \tag{14}$$

Binding borrowing constraint *If the borrowing constraint binds in period 2, the solution is*

$$b_1^{\theta *} = \frac{1}{2} b_0 + \frac{1}{2}\left[1 - (1+\kappa)\rho\right] y_1 - \frac{1}{2}(1+\kappa)\rho\theta\varepsilon_1, \qquad b_2^{\theta *} = -\kappa y_2.$$

Crisis probability *When $\theta < \frac{1-\rho+2\kappa}{\rho}$, the borrowing constraint binds if and only if*

$$\varepsilon_2 < \bar{\varepsilon}_2^\theta \equiv \frac{b_1 + (1-\rho+2\kappa)\rho y_1}{\rho\theta - (1-\rho+2\kappa)}, \qquad \textit{where } \bar{\varepsilon}_2^\theta < 0.$$

The probability of crisis is given by $\Pr(\varepsilon_2 < \bar{\varepsilon}_2^\theta) = F(\bar{\varepsilon}_2^\theta)$ *and decreases in the degree of diagnosticity θ:*

$$\frac{\partial \Pr(\varepsilon_2 < \bar{\varepsilon}_2^\theta)}{\partial \theta} < 0. \tag{15}$$

Conversely, when $\theta > \frac{1-\rho+2\kappa}{\rho}$, the borrowing constraint binds if and only if $\varepsilon_2 > \bar{\varepsilon}_2^\theta$ where $\bar{\varepsilon}_2^\theta > 0$.

The probability of crisis is given by $\Pr(\varepsilon_2 > \bar{\varepsilon}_2^{\theta}) = 1 - F(\bar{\varepsilon}_2^{\theta})$ *and increases in* θ*:*

$$\frac{\partial \Pr(\varepsilon_2 > \bar{\varepsilon}_2^{\theta})}{\partial \theta} > 0. \tag{16}$$

Proof. See Appendix A.2. ■

Allocations, defined in the equilibrium under DE described above, are not efficient from the perspective of the social planner who forms RE. Recall the second-period saving decision:

$$b_2^{\theta *} = \frac{1}{2} b_1 + \frac{1}{2}(1-\rho) y_2 - \frac{1}{2}\rho\theta\varepsilon_2.$$

The DE agents' saving ($b_2^{\theta *}$) decreases in response to the positive income news ($\varepsilon_2 > 0$). In contrast, the RE agent does not respond to the income news conditional on the observed income level y_2, as shown in (7). Since the RE solution constitutes the constrained efficient allocations, the DE agent overborrows (borrows more than the efficient level) in response to the positive income news. Overborrowing occurs because the DE agent overestimates the increase in lifetime income and boosts consumption. Conversely, in response to the negative income news ($\varepsilon_2 < 0$), the DE agent underborrows. The size of the inefficiency (overborrowing and underborrowing) increases with the degree of diagnosticity (θ) and the persistence of income (ρ).

Unlike our main assumption of DE on exogenous variables, L'Huillier et al. (2024) and Bianchi et al. (2024a) assume that agents form DE on endogenous variables. Specifically, agents form DE on an endogenous variable (consumption) instead of the exogenous variable (endowment). We also illustrate the solutions under DE on endogenous variables for comparison. Under DE on endogenous variables, the DE operator on consumption is given by:

$$E_t^{\theta}(c_{t+1}) = E_t(c_{t+1}) + \theta[E_t(c_{t+1}) - E_{t-1}(c_{t+1})], \qquad \theta \geq 0 \tag{17}$$

The following proposition then provides the associated solutions.

Proposition 3. *The equilibrium under DE on endogenous variables is characterized as follows.*

Non-binding borrowing constraint *If the borrowing constraint in period 2 does not bind, the solution is*

$$\tilde{b}_1^{\theta *} = \frac{2}{3} b_0 + \frac{2}{3}\left(1 - \frac{\rho(1+\rho)}{2}\right) y_1 - \frac{2\theta}{3(3+\theta)}(1 + \rho(1+\rho))\varepsilon_1, \tag{18}$$

$$\tilde{b}_2^{\theta *} = \frac{1}{2} b_1 + \frac{1}{2}(1-\rho) y_2 - \frac{\theta}{2(2+\theta)}(1+\rho)\varepsilon_2. \tag{19}$$

Binding borrowing constraint *If the borrowing constraint binds in period 2, the solution is*

$$\tilde{b}_1^{\theta *} = \frac{1}{2}b_0 + \Big(\frac{1-(1+\kappa)\rho}{2}\Big)y_1 - \frac{\theta}{2(2+\theta)}(1+(1+\kappa)\rho)\varepsilon_1, \tag{20}$$

$$\tilde{b}_2^{\theta *} = -\kappa y_2. \tag{21}$$

Crisis Probability *Suppose $\rho > \kappa$. When $\theta < \frac{1-\rho+2\kappa}{\rho-\kappa}$, the borrowing constraint binds if and only if*

$$\varepsilon_2 < \tilde{\bar{\varepsilon}}_2^{\theta} \equiv \frac{b_1 + (1-\rho+2\kappa)\rho y_1}{(\frac{\theta}{2+\theta})(1+\rho) - (1-\rho+2\kappa)}. \tag{22}$$

The probability of crisis is given by $\Pr(\varepsilon_2 < \tilde{\bar{\varepsilon}}_2^{\theta}) = F(\tilde{\bar{\varepsilon}}_2^{\theta})$ *and decreases in the degree of diagnosticity θ:*

$$\frac{\partial \Pr(\varepsilon_2 < \tilde{\bar{\varepsilon}}_2^{\theta})}{\partial \theta} < 0. \tag{23}$$

Conversely, when $\theta > \frac{1-\rho+2\kappa}{\rho-\kappa}$, the borrowing constraint binds if and only if $\varepsilon_2 > \tilde{\bar{\varepsilon}}_2^{\theta}$.

The probability of crisis is given by $\Pr(\varepsilon_2 > \tilde{\bar{\varepsilon}}_2^{\theta}) = 1 - F(\tilde{\bar{\varepsilon}}_2^{\theta})$ *and increases in θ:*

$$\frac{\partial \Pr(\varepsilon_2 > \tilde{\bar{\varepsilon}}_2^{\theta})}{\partial \theta} > 0. \tag{24}$$

Proof. See Appendix A.3. ■

We find that DE on exogenous variables generates overborrowing in response to a positive realized news. This occurs as long as the persistence of the income process (ρ) is greater than zero. In the setting with DE on endogenous variables, we also find overborrowing when the positive news is arrived. Crucially, however, overborrowing persists even with zero persistence ($\rho = 0$), as endogenous extrapolation—similar to the mechanisms described in L'Huillier et al. (2024) and Bianchi et al. (2024a)—is in effect.

Figure 1 graphically illustrates the quantitatively different dynamics under DE on exogenous versus endogenous variables. We compare the borrowing coefficients in response to an income news (ε_2) in (14) and (19). In panel (a), we find that overborrowing responses are stronger under DE on exogenous variables for a reasonable set of parameters for income persistence (ρ) and diagnosticity (θ). As long as ρ and θ are large enough, responses under the DE on exogenous variables are stronger than those under the DE on endogenous variables. Consequently, panel (b) shows that the crisis probability (the probability of the borrowing constraint binding) is further amplified under DE on exogenous variables as diagnosticity (θ) increases above the threshold level (θ^*), for a given level of income persistence (ρ). The threshold level can be around 1.05 and 1.75 for DE on exogenous and endogenous variables, respectively, depending on the model parameterization.

Intuitively, agents overborrow in response to good news, which increases the probability of hitting the borrowing constraint. This mechanism is stronger under DE on exogenous variables. In contrast, agents under DE on endogenous variables expect higher future consumption after receiving good news. This expectation leads to increased savings and thus mutes the positive relationship between diagnosticity and crisis probability. Interestingly, below the threshold level of diagnosticity (θ^*), the crisis probability decreases with θ. This occurs because a higher degree of diagnosticity also intensifies the underborrowing driven by pessimism (when $\varepsilon_2 < 0$). In regions far from the borrowing constraint, this effect serves to reduce the overall probability of the constraint binding. As a result, the relationship between the crisis probability and diagnosticity changes depending on the relative forces of overborrowing and underborrowing.

Additionally, we find that the crisis probability is lower under DE than under the RE benchmark ($\theta = 0$) for values of θ around the threshold level. Maxted (2024) also shows that, within a baseline calibration of a financial intermediary model, the crisis probability is lower under DE than under RE. The underlying intuition is that diagnostic pessimism prompts intermediaries to accumulate larger equity buffers, which effectively reduces the likelihood of a systemic crisis. Our model nests the results of Maxted (2024) for values of θ around the threshold θ^*.

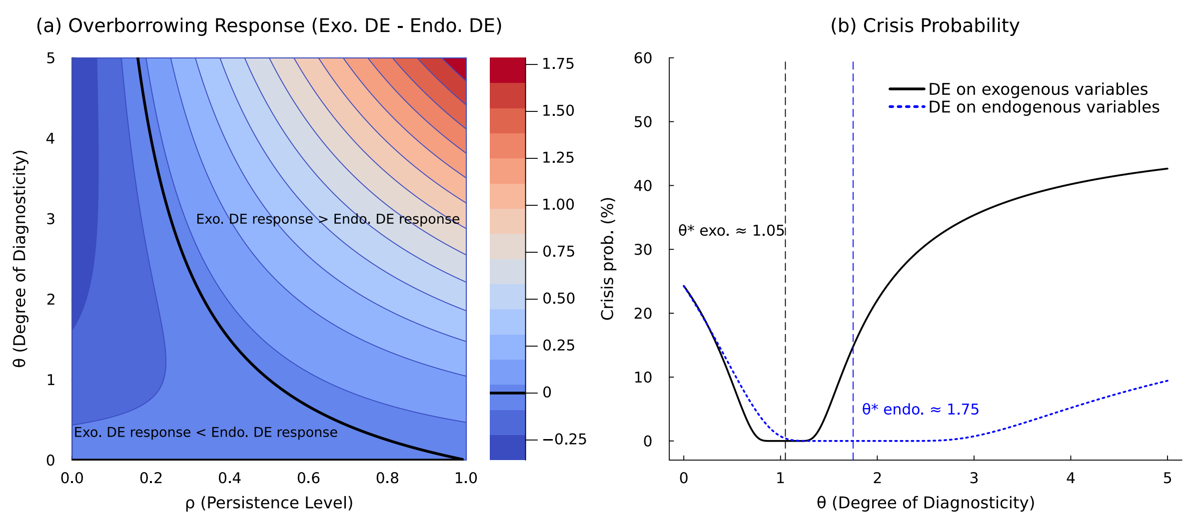

Figure 1: Overborrowing Response and Crisis Probability

Notes: Panel (a) plots the difference between the absolute overborrowing response coefficients for DE on exogenous versus endogenous variables, $|\alpha^{\theta}_{\varepsilon,2}| - |\tilde{\alpha}^{\theta}_{\varepsilon,2}| = \frac{1}{2}\rho\theta - \frac{\theta}{2(2+\theta)}(1+\rho)$. In the region above the bold black line, this difference is positive, indicating a larger response under DE on exogenous variables. Panel (b) presents the crisis probability–the probability of the borrowing constraint binding–across different degrees of diagnosticity (θ). Panel (b) uses the following parameters: $\rho = 0.80$, $\kappa = 0.32$, $b_1 = 0.5$, $y_1 = 1$, and $\sigma = 1$.

3 A Small Open Economy Model with Diagnostic Expectations

3.1 Model Setup

We consider the two-sector small open economy (SOE) model proposed by Mendoza (2002) and Bianchi (2011).[8] The only substantial difference relative to the typical model for analyzing Sudden Stops events is that a representative household forms her expectations *diagnostically*.

The economy is populated by a continuum of identical, infinitely-lived households of measure unity. Each household consumes the composite of tradable and non-tradable goods to maximize her expected lifetime utility:

$$u(c_t) + E_t^{\theta}\left[\sum_{s=1}^{\infty} \beta^s u(c_{t+s})\right], \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{25}$$

where $\beta \in (0,1)$ is the discount factor, and $\gamma > 0$ is the coefficient of relative risk aversion. The consumption basket c_t is a CES (constant elasticity of substitution) aggregator between tradable c_t^T and non-tradable goods c_t^N given by:

$$c_t = \left[\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}\right]^{-\frac{1}{\eta}}, \quad \eta > -1,\ \omega \in (0,1), \tag{26}$$

where ω is the proportion of tradable goods in total consumption, and the elasticity of substitution is $1/(1+\eta)$. In each period t, households receive a stochastic endowment of tradables y_t^T and a fixed endowment of non-tradables y^N.[9] The log of y_t^T follows a first-order Markov process given by:

$$\log y_t^T = \rho_T \log y_{t-1}^T + \varepsilon_t^T. \tag{27}$$

where $\rho_T \in (0,1)$ is the persistence level of tradable endowment process, and ε_t^T is Gaussian with a mean zero and a standard deviation σ_T. The agent with a DE constructs a distorted belief on the future evolution of y_t^T. By equation (4), the perceived process of a diagnostic agent can be described as a lognormal process:

$$\log y_{t+1}^T \mid (\log y_t^T, \varepsilon_t^T) \sim N\big(\rho_T \log y_t^T + \theta\rho_T\varepsilon_t^T,\ \sigma_T^2\big). \tag{28}$$

when $\theta > 0$, the agent predicts their future tradable endowment by overweighting current realized news as long as ρ_T is greater than zero. As a result, current news enters the diagnos-

[8] We also extend the model to incorporate production in Appendix C.

[9] Unlike Bianchi (2011), we assume that the non-tradable endowment are fixed for computational tractability. However, we find that this assumption does not alter the quantitative feature of the model as in Bianchi et al. (2016) and Arce et al. (2019). See Appendix H for details.

tic household's decision problem as a state variable, distinguishing our approach from that of Bianchi (2011). This difference highlights the model's novel approach to considering the impact of current information, or "news," on household borrowing decisions along with the occasionally binding constraint.[10]

The representative agent borrows through a non-state contingent foreign bond, which is denominated in tradable goods units and carries a fixed interest rate r, set exogenously by the international bond market. By normalizing the price of tradable goods to one and denoting the price of non-tradable goods as p_t^N, the household's budget constraint can be expressed as follows:

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1+r) + y_t^T + p_t^N y_t^N. \tag{29}$$

where b_{t+1} is the bond holdings that the household chooses at the start of period t. Following the convention of the literature, positive values of b indicate that the household holds positive net assets, while negative values imply that the household holds debt exceeding the level of the assets.

The amount of debt cannot exceed the sum of a κ^T fraction of tradable income and a κ^N fraction of non-tradable income. This constraint ensures that the amount of borrowing is constrained by a proportion of the respective incomes from both tradable and non-tradable sectors:

$$b_{t+1} \geq -(\kappa^N p_t^N y_t^N + \kappa^T y_t^T). \tag{30}$$

The borrowing constraint captures two key empirical facts. First, Jappelli (1990) shows that the crucial determinant of credit market accessibility is the current income using the survey data. Second, in emerging markets, external debt often takes the form of short-term contracts which is denominated in a foreign currency, effectively aligning with the fact that debt is denominated in the unit of tradable goods in our framework.[11]

3.2 Household's Problem

The diagnostic households make their decisions considering three state variables: current tradable income y^T, the realized tradable income news ε^T, and their bond holdings b. We collect

[10] In the previous literature incorporating "news shock," current news shocks have a direct impact on future endowment levels or signal a permanent component in endowments (Boz (2009), Bianchi et al. (2016), Herreño and Rondón-Moreno (2025), and Flemming et al. (2019)). However, in our model, current news only distorts individual beliefs based on representative states, rather than directly affecting the future endowment level or information set.

[11] Ottonello et al. (2022) show that both models with current income and future income exhibit similar aggregate dynamics. See Drechsel and Kim (2024) for policy implications from models with alternative borrowing constraints: the earnings-based (debt-to-income) constraint and the asset-based (debt-to-asset) constraint.

all of the exogenous state variables in the vector $s = (y^T, \varepsilon^T)$. Following Bordalo et al. (2021), we focus on the recursive equilibrium under the DE.[12] Then, the maximization problem is as follows:

$$V^{\theta,ce}(s,b) = \max_{b',c^T,c^N} u(c(c^T,c^N)) + \beta E^{\theta}\left[V^{\theta,ce}(s',b')|s\right] \tag{31}$$

subject to

$$b' + p^N c^N + c^T = y^T + (1+r)b + p^N y^N,$$

$$b' \geq -(\kappa^N p^N y^N + \kappa^T y^T),$$

where variables without a superscript refer to the current period, while those with a prime superscript ($'$) denote the next period. Unlike the model presented in Bianchi (2011), in our framework, individual households choose their consumption and borrowing decisions based on both current tradable income y^T and the news on current tradable income ε^T. Our model includes ε^T as a state variable, since the perceived distribution of future tradable income $y^{T\prime}$ depends on the realized news. This formulation accommodates the DE, which have distinctive implications for how households respond to the recent news. Then, the competitive equilibrium under DE is defined as follows.

Definition 1. *(Diagnostic Competitive Equilibrium) A diagnostic competitive equilibrium is a collection of prices $\{p^N, r\}$ and decision rules $\{b'(s,b), c^T(s,b), c^N(s,b)\}$ with associated value function $V^{\theta,ce}(s,b)$ such that:*

1. *Given prices, the decision rules $\{b'(s,b), c^T(s,b), c^N(s,b)\}$ and value function $V^{\theta,ce}(s,b)$ solve the recursive optimization problem in (31) under the DE.*
2. *Markets clear: $c^N(s,b) = y^N$ and $c^T(s,b) = y^T + b(1+r) - b'(s,b)$.*
3. *Expectations of households E^{θ} are consistent with the dynamics governed by the perceived income process (28).*

To enhance our understanding of how borrowing constraints impact a competitive equilibrium, we focus on the solution in the sequential form. The first-order conditions are:

$$\lambda_t = u_T(t)^{13}, \tag{32}$$

[12]Under DE on exogenous variables, the sequential problem is identical to the recursive problem as we show in Appendix B.1. Moreover, the recursive formation has an advantage in that we can focus on the Markov stationary equilibrium where social planner's policy is time-consistent as shown in Bianchi and Mendoza (2018).

[13]$u_T(t) = u'(c_t)\left[\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}\right]^{-\frac{1}{\eta}-1}\omega(c_t^T)^{-\eta-1}$

$$p_t^N = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1}, \tag{33}$$

$$\lambda_t = \beta(1+r)E_t^\theta \lambda_{t+1} + \mu_t, \tag{34}$$

$$b_{t+1} + (\kappa^N p_t^N y_t^N + \kappa^T y_t^T) \geq 0, \qquad \text{with equality if } \mu_t > 0, \tag{35}$$

where λ is the Lagrange multiplier associated with the budget constraint and μ is the Lagrange multiplier associated with the borrowing constraint. The optimality condition (33) aligns the marginal rate of substitution between tradable goods and non-tradable goods with their relative price. This equilibrium condition implies that changes in c^T will influence the borrowing constraint by altering the price of collateral, p^N. The pecuniary externality arises because private agents do not internalize the effect of their decisions on equilibrium prices. Condition (34) represents the Euler equation in which the marginal benefit of additional borrowing in the current period needs to be equalized to the marginal cost of borrowing, including interest payments and the possibility of hitting the borrowing constraint in the future period. The DE channel mainly works through equation (34) by affecting the agent's borrowing decision and the associated consumption path. Equation (35) is the complementary slackness condition regarding the borrowing constraint.

3.3 Social Planner's Problem

In contrast to individual households, the social planner is constrained in the sense that she lets the non-tradable goods market clear competitively, while the planner internalizes the effects of bond holdings on the non-tradable good price. Furthermore, within this framework, we consider two types of social planners who could solve the optimization problem under either RE or DE. We can conceptualize the rational social planner as a "paternalistic policymaker" who understands the true distribution of the stochastic process and corrects diagnostic households' distorted beliefs. Conversely, the diagnostic social planner, interpreted as a "non-paternalistic policymaker," shares the same distorted beliefs about future events with households. Consequently, this belief disagreement between the rational social planner and diagnostic households leads to two significant implications for welfare. First, diagnostic households, becoming over-optimistic in response to positive economic news, may experience a reduction in welfare compared to the optimal scenario anticipated under RE. Secondly, this disagreement may intensify the pecuniary externalities, as it encourages households to increase their borrowing in response to positive news. We will analyze these implications quantitatively in the following section.

The recursive problem of a diagnostic social planner can be expressed by:

$$V^{\theta,sp}(s,b) = \max_{b',c^T} u(c(c^T,y^N)) + \beta E^{\theta}\left[V^{\theta,sp}(s',b')|s\right] \tag{36}$$

subject to

$$b' + c^T = y^T + (1+r)b,$$

$$b' \geq -\left(\kappa^N \frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + \kappa^T y^T\right),$$

where $s = (y^T, \varepsilon^T)$ is a vector that contains the current tradable income and realized tradable income news. The equation (33) determining the price of non-tradable goods p^N is plugged in the borrowing constraint, as the social planner internalizes the effects of aggregate borrowing on the price. Similar to diagnostic households, a diagnostic social planner makes consumption and borrowing decisions based on both the level of tradable income and news about current tradable income. It is important to note that, unlike the diagnostic planner, a rational social planner does not take the realized news into consideration. A rational social planner's problem is defined by the following recursive form:

$$V^{sp}(y^T,b) = \max_{b',c^T} u(c(c^T,y^N)) + \beta E\left[V^{sp}(y^{T\prime},b')|y^T\right] \tag{37}$$

subject to

$$b' + c^T = y^T + (1+r)b,$$

$$b' \geq -\left(\kappa^N \frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + \kappa^T y^T\right).$$

To emphasize the differences between the household and a constrained social planner, we employ the sequential form of equations (36) and (37). The superscript "sp" is used to differentiate the Lagrange multipliers of the social planner's problem from those in the diagnostic competitive equilibrium, while θ is utilized to distinguish between the Lagrange multipliers of the diagnostic and rational social planners. The first-order conditions from the sequential problem of the diagnostic social planner are:

$$\lambda_t^{\theta,sp} = u_T(t) + \mu_t^{\theta,sp}\psi_t^{\theta}, \tag{38}$$

$$\lambda_t^{\theta,sp} = \beta(1+r)E_t^{\theta}\lambda_{t+1}^{\theta,sp} + \mu_t^{\theta,sp}, \tag{39}$$

$$b_{t+1} + \left(\kappa^N \frac{1-\omega}{\omega}\left(\frac{c_t^T}{y_t^N}\right)^{\eta+1} y_t^N + \kappa^T y_t^T\right) \geq 0, \qquad \text{with equality if } \mu_t^{\theta,sp} > 0, \tag{40}$$

where $\lambda^{\theta,sp}$ is the Lagrange multiplier associated with the budget constraint, and $\mu^{\theta,sp}$ is the Lagrange multiplier associated with the borrowing constraint for the diagnostic social planner. The first-order conditions of the rational social planner are:

$$\lambda_t^{sp} = u_T(t) + \mu_t^{sp}\psi_t, \tag{41}$$

$$\lambda_t^{sp} = \beta(1+r)E_t\lambda_{t+1}^{sp} + \mu_t^{sp}, \tag{42}$$

$$b_{t+1} + \left(\kappa^N \frac{1-\omega}{\omega}\left(\frac{c_t^T}{y_t^N}\right)^{\eta+1} y_t^N + \kappa^T y_t^T\right) \geq 0, \qquad \text{with equality if } \mu_t^{sp} > 0, \tag{43}$$

where λ^{sp} is the Lagrange multiplier associated with the budget constraint, and μ^{sp} is the Lagrange multiplier associated with the credit constraint for the rational social planner. The pecuniary externality term $\psi = \kappa^N(\eta+1)\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{y^N}\right)^\eta$ captures the effects of tradable consumption on the non-tradable good price and thus on the borrowing capacity. Even if the credit constraint is not binding today ($\mu_t^{sp} = 0$), these effects work through equation (42). To see concisely, suppose that the borrowing constraint does not bind today ($\mu_t^{sp} = 0$), and it binds tomorrow $\mu_{t+1}^{sp} > 0$. Then, combining equation (41) and (42), we have the following Euler equation for the rational social planner:

$$u_T(t) = \beta(1+r)E_t\left[u_T(t+1) + \mu_{t+1}^{sp}\psi_{t+1}\right]. \tag{44}$$

In equation (44), the term $\mu_{t+1}^{sp}\psi_{t+1}$ captures the likelihood of a binding credit constraint tomorrow, which leads the social planner to borrow less than the household does. The higher expectations of a binding credit constraint, denoted by $E_t\left[\mu_{t+1}^{sp}\right] > 0$, translates into a higher marginal cost of borrowing perceived by the social planner.

Our DE problem in its recursive formulation is time-consistent. Following the approach of Klein et al. (2008) and Klein et al. (2007), we analyze Markov stationary policy rules, which depend on the state variables (b, s). Since the social planner lacks commitment power, it sets the current policy assuming the future social planner's rules are given. A Markov perfect equilibrium occurs when the policy rules assumed for future planners align with what the current planner finds optimal. This consistency means that the planner has no incentive to deviate, making the rules time-consistent. Similarly, Bianchi and Mendoza (2018) find that optimal policy rules are time-consistent under their recursive formulation.

While the rational social planner does not overreact to realized news, the diagnostic planner and households become overly optimistic about positive news and overly pessimistic about negative ones. Thus, as in Schmitt-Grohé and Uribe (2021), individual households may borrow less than the rational planner following a negative news. This occurs because the diagnostic household's pessimistic outlook amplifies precautionary-saving motives and lowers expected future income, leading to reduced borrowing relative to the rational benchmark.

4 Optimal Macroprudential Policy

To address both pecuniary externalities and belief distortions, we consider the implementation of a tax on foreign debt (equivalent to capital controls). The optimal tax is characterized as a distortionary instrument designed to align the household's Euler equation with that of the social planner. When the social planner's credit constraint binds in period t, we set the tax to zero, as the allocations of the planner and the diagnostic household coincide (see Bianchi, 2011). Notably, the optimal tax schedule for a diagnostic social planner differs from that of a rational social planner; whereas the rational planner corrects for discrepancies in future income expectations, the diagnostic planner does not, as demonstrated in the following propositions.

Proposition 4. *(Optimal debt tax of the non-paternalistic policymaker) In states where the borrowing constraint of the diagnostic social planner is not binding (i.e., $\mu_t^{sp,\theta} = 0$), the constrained-efficient allocations of the diagnostic social planner can be decentralized as a diagnostic competitive equilibrium with the following optimal debt tax:*

$$\tau_t^{\theta,sp} = \frac{E_t^\theta\left[\mu_{t+1}^{\theta,sp}\psi_{t+1}^\theta\right]}{E_t^\theta\left[u_T(t+1)\right]}. \tag{45}$$

Proof. See Appendix B.2. ■

Proposition 5. *(Optimal debt tax of the paternalistic policymaker) In states where the borrowing constraint of the rational social planner is not binding (i.e., $\mu_t^{sp} = 0$), the constrained-efficient allocations of the rational social planner can be decentralized as a diagnostic competitive equilibrium with the following optimal debt tax:*

$$\tau_t^{sp} = \frac{E_t\left[u_T(t+1) + \mu_{t+1}^{sp}\psi_{t+1}\right]}{E_t^\theta\left[u_T(t+1)\right]} - 1. \tag{46}$$

Proof. See Appendix B.3. ■

As a result of the optimal tax schedule, we will show that the rational social planner may subsidize ($\tau_t^{sp} < 0$) the diagnostic household to borrow more when bad news arrives, a result that is distinct from the findings of the previous literature (Bianchi, 2011, Bianchi et al., 2016, Herreño and Rondón-Moreno, 2025).

To more precisely examine how the rational social planner's taxing behavior varies when facing diagnostic versus rational households, we define the optimal tax schedule for the latter case as the benchmark, following Bianchi (2011).[14] This benchmark tax schedule induces households to internalize the effect of their borrowing on the relative price of non-tradables and is characterized by:

$$\tau_t^{\star,sp} = \frac{E_t\left[\mu_{t+1}^{sp}\psi_{t+1}\right]}{E_t\left[u_T(t+1)\right]}. \tag{47}$$

Consequently, the optimal debt tax for a paternalistic policymaker is representable as the sum of two distinct components. This allows for a clear distinction between policy interventions aimed at the pecuniary externality and those addressing expectation misalignments, as well as the resulting interaction effects.

Proposition 6. *(Optimal debt tax decomposition) In states where the borrowing constraint of the rational social planner is not binding (i.e., $\mu_t^{sp} = 0$), the optimal debt tax of the paternalistic policymaker can be expressed as*

$$\tau_t^{sp} = \underbrace{\frac{E_t[u_T(t+1)]}{E_t^{\theta}[u_T(t+1)]}\tau_t^{\star,sp}}_{\text{PE-DE interaction corrected}} + \underbrace{\frac{E_t[u_T(t+1)] - E_t^{\theta}[u_T(t+1)]}{E_t^{\theta}[u_T(t+1)]}}_{\text{DE corrected}}. \tag{48}$$

Proof. See Appendix B.4. ■

As with the "information term" in the decomposed debt tax in Bianchi et al. (2012), the second term in (48) arising from belief differences vanishes when private agents form expectations identically to the planner. In our framework, extrapolative beliefs about future income can either attenuate or amplify the externality term.

Specifically, the rational social planner corrects the borrowing decisions of diagnostic households by adjusting for both the pecuniary externality, $\mu_{t+1}^{sp}\psi_{t+1}$, and the discrepancy between rational and diagnostic expectations. When households overreact to favorable news, the rational social planner implements a debt tax, τ_t^{sp}, that mitigates overborrowing more aggressively than the benchmark tax schedule, $\tau_t^{\star,sp}$, which would be implemented for rational households.

[14]By construction, the objective of the optimal macroprudential tax is to implement the allocations that a rational social planner would choose in the problem (37) for a decentralized economy populated by diagnostic households. The allocations chosen by the rational social planner with the diagnostic households correspond to the constrained Pareto optimum in Bianchi (2011). Thus, the social planner's Lagrangian multiplier coincides with the multiplier from Bianchi's framework.

Table 1: Baseline Parameters

Parameter	Description	Value
r	Interest rate	0.04
γ	Relative risk aversion	2
$1/(1+\eta)$	Elasticity of substitution	0.83
ω	Share of tradable goods	0.31
β	Discount factor	0.91
$\kappa^T = \kappa^N$	Credit coefficient	0.323
y^N	Non-tradable endowment	1
θ	Diagnosticity parameter	1.415

This result follows directly from (48): since optimism following the positive income news raises consumption expectations and lowers the expected marginal utility of consumption, the planner offsets this inefficiency by increasing the optimal tax. This adjustment is driven by two factors: first, the interaction of $\tau_t^{\star,sp}$ with the ratio of expected marginal utilities, and second, the wedge created by the disparity between these expectations. In the subsequent quantitative analysis, we decompose and quantify the contribution of each term. Conversely, when negative news are realized, diagnostic households may underborrow due to excessive pessimism, necessitating a subsidy from the rational planner. Our numerical results in the following section confirm that the optimal debt tax rule indeed turns negative in such states.

5 Quantitative Analysis

We now calibrate the model to investigate business cycles and crisis dynamics under DE. We also explore the optimal tax schedule and assess the performance of simple policy rules including fixed taxes and macroprudential Taylor rules (Bianchi and Mendoza, 2018) to provide more practical policy insights.

5.1 Calibration and Solution Method

Our model is calibrated to match data from Argentina, which is adopted from Bianchi (2011). The baseline parameters of the model are shown in Table 1.

The annual interest rate of international risk-free bonds is set to 4 percent and the coefficient of relative risk aversion is set to $\gamma = 2$, which are standard values in the literature. As Bianchi (2011) discussed, the intratemporal elasticity of substitution is the crucial parameter because it determines the magnitude of the pecuniary externality by affecting the price of non-tradable goods in equation (33). We set the $1/(1+\eta) = 0.83$ which is aligning with Bianchi (2011) and Bianchi et al. (2016). The share of tradable goods is set to 0.31. The remaining parameters β and κ^T are used to match the long-run moments of the Argentina data. We select $\beta = 0.91$ and $\kappa^T = 0.323$ to ensure that the diagnostic competitive equilibrium aligns with net foreign assets to GDP ratio (-29 percent of GDP) and a financial crisis probability of 5.5% in Argentina.

We discretize the first-order Markov process of the tradable endowment process into 21 states using the method of Tauchen (1986). Following Bianchi (2011), we set the persistence and volatility parameters to $\rho_T = 0.53$ and $\sigma_{y^T} = 0.058$, respectively. We next construct the news grid to preserve the state space of the true endowment process. The transition probabilities for future tradable income perceived by diagnostic agents, which depend on current income and news, are then discretized to capture overreactions to recent news (See Appendix D). Note that the resulting diagnostic transition matrix differs from the rational one because rational agents do not overreact to realized news.

The model is solved using a time-iteration algorithm, a global solution method following Bianchi (2011); further details are provided in Appendix D. We employ a global solution method because local approximations struggle to accurately characterize the dynamics of net foreign assets, which is an indispensable component of SOE-DSGE models, as noted by de Groot et al. (2025). While local solution methods offer computational speed, de Groot et al. (2025) demonstrate that they poorly capture precautionary saving motives, even when incorporating occasionally binding constraints. This limitation is particularly problematic in models where the accurate representation of savings behavior is crucial for understanding the dynamics of rare events, such as financial crises in emerging economies.

The diagnosticity parameter, θ, is calibrated to match the empirical magnitude of the HP-detrended credit boom preceding the Sudden Stop events identified in the dataset of Bianchi (2011). We define a Sudden Stop as an episode in which net capital outflows rise by more than one standard deviation relative to their sample mean. To match the HP-detrended current account to GDP ratio of –1.41% during the credit boom phase, we set $\theta = 1.415$, a value that lies well within the range of estimates found in the literature.[15] Figure A.8 shows that credit booms are amplified as θ increases, implying that incorporating DE is essential for the model to match the magnitude of credit booms observed in the data. Conversely, the rational-expectations framework in Bianchi (2011) struggles to account for the intensity of these credit booms. We also explore the implications of different values of θ through various sensitivity exercises in Appendix E. We find that, beyond a certain threshold of θ, the probability of crises increases, and the subsequent corrections in the real exchange rate, consumption, and capital flows are significantly

[15] Bordalo et al. (2020) show that the estimated θ varies significantly depending on the variable being predicted, with estimates ranging from 0.34 to 1.37. Furthermore, their 95% confidence interval includes values as high as 2.31.

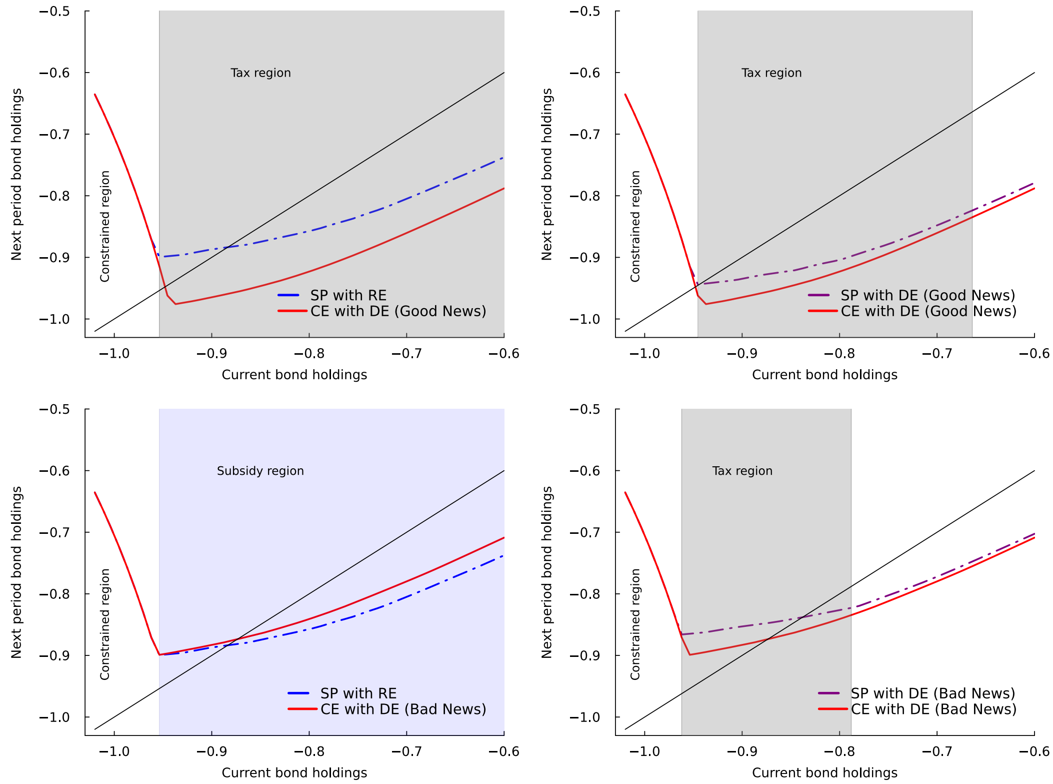

Figure 2: Bond Decision Rules in a Low Income State

Notes: This figure shows the bond decision rules in the decentralized competitive equilibrium under DE and in the constraint-efficient equilibrium when the current tradable income state is one standard deviation below the trend. The gray-shaded region indicates that the optimal debt tax is positive, and the blue-shaded region indicates that the optimal debt tax is negative (which implies that the planner subsidizes household borrowing).

amplified. In general, our results are quite robust to alternative values of the diagnosticity parameter θ. We also perform robustness tests regarding alternative parameterizations including the parameter of the borrowing constraint κ, discount factor, interest rates, persistence and volatility of the income process in Appendix F.

5.2 Borrowing Decision Rules

Figure 2 presents borrowing decision rules in the decentralized competitive equilibrium and the constrained-efficient equilibrium. We plot these decision rules for the states of good news (upper panels) and bad news (lower panels), and the state of tradable income is one standard deviation below the trend (low income states). We also show the decision rules of the social planner with both RE (SP with RE, left panels) and DE (SP with DE, right panels) in the constrained-efficient equilibrium. In both right and left panels, households in the competitive equilibrium are under DE (CE with DE).

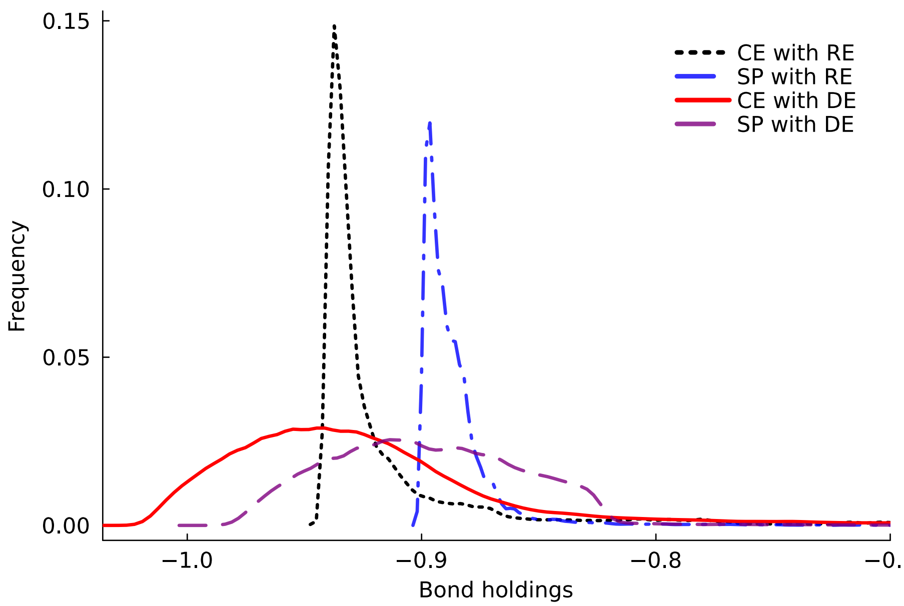

Figure 3: Ergodic Distributions of Bond Holdings
Notes: The ergodic distributions are computed by simulating one million stochastic time-series data, using the policy function of each equilibrium.

Following favorable news, a rational social planner (RE) chooses a lower level of next-period borrowing—or, equivalently, higher bond holdings—than diagnostic households (DE), for a given level of current assets (upper-left panel). This divergence indicates that diagnostic households overborrow relative to the rational benchmark, justifying the implementation of macroprudential debt taxes. As current bond holdings fall below a specific threshold, the borrowing constraint binds, necessitating deleveraging. Conversely, in response to negative news, a subsidy region emerges where the rational social planner's optimal borrowing exceeds that of diagnostic households (lower-left panel). Despite the pecuniary externality that typically induces overborrowing, the rational planner incentivizes diagnostic agents to increase borrowing, as their over-pessimism leads to underborrowing relative to the constrained-efficient level.

The existence of this underborrowing (subsidy) region distinguishes our framework from that of Bianchi (2011). Notably, as shown in the right panels, the subsidy region disappears when comparing the diagnostic social planner with diagnostic households. In that case, because the planner and households share the same distorted expectations, the planner focuses solely on correcting the pecuniary externality, which consistently leads to overborrowing.

Figure 3 presents the ergodic distributions of bond holdings, computed using the decision rules obtained under various equilibria. To derive these distributions, we simulate the model for one million periods. The competitive equilibrium with diagnostic households (CE with DE) exhibits a more dispersed distribution of borrowing than the rational benchmark (CE with RE), as the DE borrowing rule varies significantly with the realized news state. Furthermore, as reported in Table 2, the mean leverage ratio ($-b/y$) is higher under DE than under RE (29.69% vs. 29.32%).

This elevated leverage suggests that DE amplifies overborrowing by inducing households to "neglect the risk" of the borrowing constraint binding. This neglect reduces the subjective probability of a crisis, leading to a substantial increase in borrowing following favorable news. While diagnostic agents overstate tail risks upon receiving bad news, the effect of risk neglect during booms outweighs the impact of over-pessimism during busts. This asymmetry arises from the nature of the occasionally binding collateral constraint: the expansion of debt during optimistic periods is quantitatively more prolonged and significant than the sharp contraction during pessimistic episodes. This mechanism is reminiscent of Bordalo et al. (2018), where the neglect of risk acts as a precursor to financial crises.

The rational social planner (SP with RE) chooses, on average, to borrow less than households in the competitive equilibrium, resulting in an ergodic distribution that is substantially skewed toward higher bond holdings. The diagnostic social planner (SP with DE) also prefers lower leverage than the competitive equilibrium households. However, its bond distribution is far more dispersed than that of the rational planner, as the diagnostic planner's decision rule remains sensitive to the current news state, leading to fluctuating target debt levels.

5.3 Financial Crises Analysis

Figure 4 plots the average aggregate dynamics surrounding financial crises. Within a simulation of one million periods, a financial crisis event is identified as a period in which the credit constraint on the diagnostic household binds and the net capital outflow exceeds one standard deviation from its mean in the ergodic distribution. We then compute the average across all such events to generate the event-study dynamics.

One salient feature is that DE amplify financial crisis dynamics. In the competitive equilibrium with rational households (CE with RE), crises are characterized by a depreciation of the real exchange rate, a collapse in borrowing (reflected by a rise in bond holdings), a decline in tradable consumption, and a reversal in the current-account-to-GDP ratio. These variables revert sharply toward their means following the crisis, consistent with the stylized facts of historical Sudden Stop episodes.

When the model incorporates DE (CE with DE), the real exchange rate depreciation is more pronounced, and the credit boom-bust cycle is significantly amplified. These effects stem from the fact that diagnostic households overreact to realized news: the borrowing constraint binds

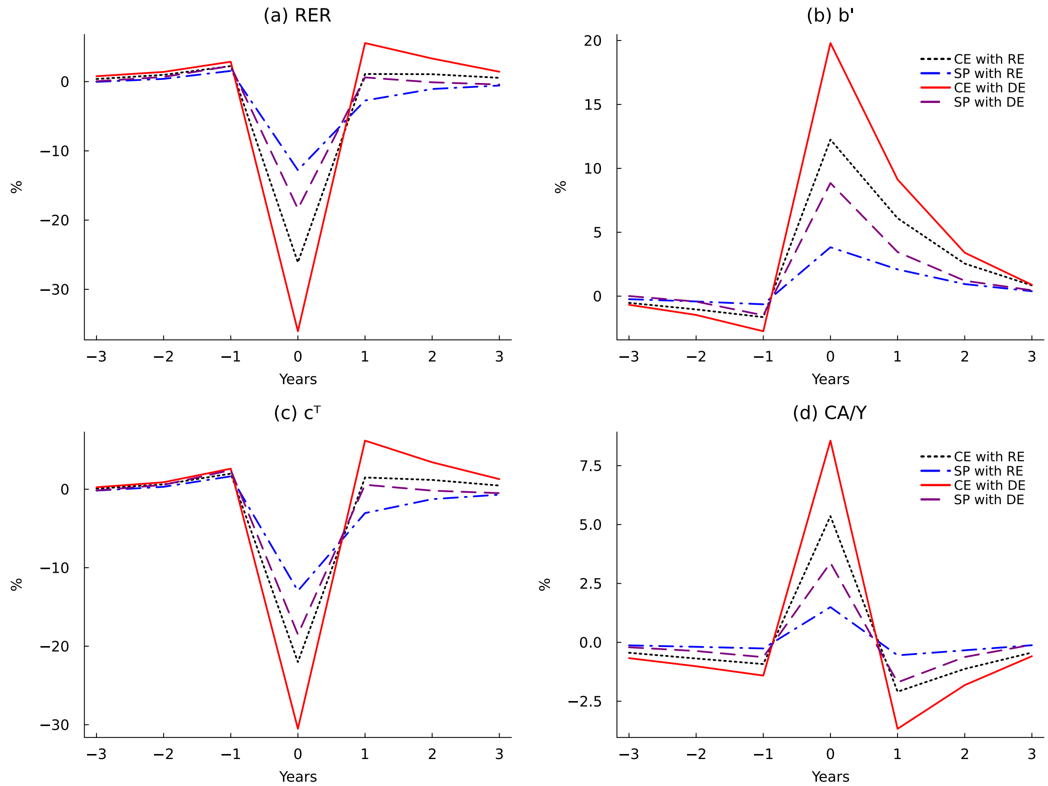

Figure 4: Crises Dynamics: Sudden Stop Events Analysis
Notes: The panels (a), (b), and (c) are plotted as percentage differences relative to the ergodic mean. Panel (d) shows HP detrended current-account to GDP ratio with a smoothing parameter 10.

more severely due to a sharp reversal in perceptions—from optimistic to pessimistic—during the crisis.

Furthermore, while the rational social planner (SP with RE) successfully mitigates these fluctuations, the diagnostic social planner (SP with DE) is less effective. Because the diagnostic planner shares the households' distorted beliefs, its capacity to tame the cycle is more limited than that of a rational policymaker, who correctly understands the true data-generating process and explicitly accounts for the underlying belief distortions.

Table 2 summarizes the business cycle moments derived from the model simulations. Under rational expectations (columns 1–2), the social planner substantially mitigates crises relative to the competitive equilibrium, reducing the crisis probability from 5.58% to 1.20% and dampening both the volatility and countercyclicality of the current account. Furthermore, the average output drop during a crisis decreases from 20.13% to 13.14%.

In contrast, the inclusion of DE significantly amplifies crisis dynamics in the competitive equilibrium (column 3). This case is characterized by more severe output contractions, sharper real exchange rate depreciations, and more intense capital outflows than the rational bench-

Table 2: Business Cycle Moments

	Rational Expectations (RE)		Diagnostic Expectations (DE)	
	(1) CE	(2) SP	(3) CE	(4) SP
A. Average Dynamics				
Mean of Leverage (%)	29.36	28.35	29.81	28.80
Crisis Probability (%)	5.58	1.20	5.53	0.99
$\sigma(CA/y)$ (%)	2.62	1.00	3.01	1.70
$\rho(y, CA/y)$	-0.70	-0.43	-0.78	-0.72
B. Crisis Dynamics				
$Consumption$ (%)	-8.03	-4.36	-11.15	-6.26
$Output$ (%)	-20.13	-13.14	-26.83	-17.60
RER Depreciation (%)	26.10	12.79	36.05	18.37
CA/y (%)	5.36	1.50	8.56	3.38

Notes: Columns (1) and (2) report business cycle moments for the competitive equilibrium (CE) with rational households and the rational social planner's allocation (SP), respectively. Columns (3) and (4) report the corresponding moments for the CE with diagnostic households and the SP with a diagnostic planner.

mark. Ultimately, the DE framework generates large real exchange rate depreciations while preserving standard business cycle moments, such as the countercyclicality of the current account and the volatility of capital flows. The diagnostic social planner (column 4) also alleviates these fluctuations, though less effectively than the rational planner (column 2). For instance, the output drop of the diagnostic planner is approximately 6.26%. Although this is an improvement over both competitive equilibria, it exceeds the decline under the rational social planner.

5.4 Optimal Macroprudential Policy and Welfare Gains

This section examines the implementation of optimal macroprudential policy and its associated welfare implications. As characterized in the previous section, the optimal debt tax is imposed on household borrowing to achieve constrained-efficient allocations.

Figure 5 presents the optimal debt tax schedules required to achieve constrained-efficient allocations under both the rational social planner (SP with RE, the paternalistic planner) and the diagnostic social planner (SP with DE, the non-paternalistic planner). We focus on a state of low tradable income, defined as one standard deviation below the trend.

In the case of favorable news (upper-left panel), the paternalistic policymaker implements a debt tax to align allocations with the rational social planner's target. The optimal tax rate increases as current bond holdings decline (i.e., as debt rises), reflecting the higher marginal benefit of reducing the probability that the borrowing constraint binds. Consistent with Bianchi (2011), the tax is set to zero once debt reaches the threshold at which the constraint binds in the current period.

Following negative news (lower-left panel), the paternalistic policymaker's strategy involves subsidies. In the subsidy region (indicated by the blue shaded area), the policymaker incentivizes borrowing to counteract the underborrowing driven by diagnostic households' excessive pessimism. However, these subsidies are phased out as debt increases toward the threshold, as further subsidization would inadvertently trigger the borrowing constraint.

This finding stands in sharp contrast to the baseline Bianchi (2011) model with rational expectations, which only exhibits overborrowing under standard parameterizations (Table 1). While Schmitt-Grohé and Uribe (2021) demonstrate that underborrowing can occur in the Bianchi model under alternative parameters, our model endogenously generates both overborrowing and underborrowing regions within a single parameterization, depending on the realization of

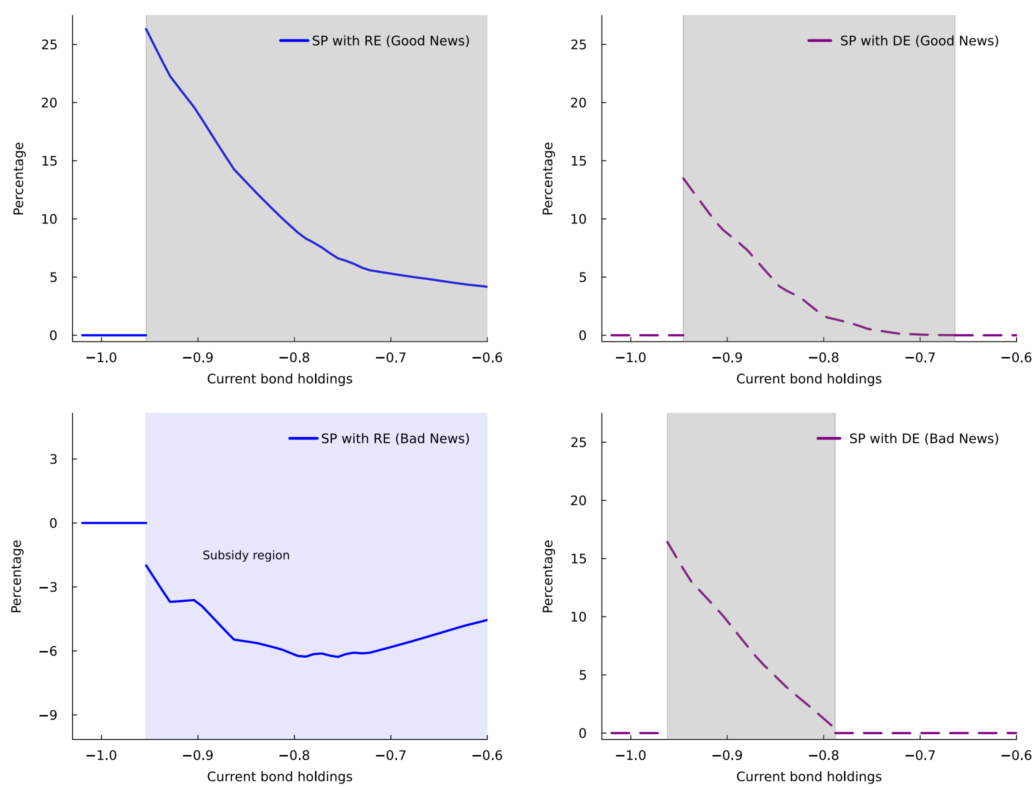

Figure 5: Optimal Debt Tax Schedule

Notes: This figure shows the optimal tax schedule of the paternalistic planner (SP with RE) and the non-paternalistic planner (SP with DE) when the current tradable income state is one standard deviation below the trend. The gray-shaded region indicates that the optimal tax is positive, and the blue-shaded region indicates that the optimal tax is negative (which implies that the planner subsidizes household borrowing).

news.

The right column of Figure 5 displays the optimal policy of the non-paternalistic (diagnostic) social planner. While this planner also raises taxes as debt increases to internalize the pecuniary externality, it never provides subsidies. Notably, the non-paternalistic planner increases the tax rate in response to bad news, as households attempt to borrow more than the diagnostic planner, thereby intensifying the pecuniary externality. Conversely, following bad news, the paternalistic planner reduces taxes or increases subsidies in order to correct households' excessive pessimism. This divergence in policy response explains the heterogeneous cyclicality of optimal taxes: we find a positive correlation between detrended output and the optimal debt tax under paternalistic policy, but a negative correlation under non-paternalistic policy. We explore this further in Section 5.5 and Table 5.

In Section 3.3, the optimal debt tax of the paternalistic social planner (τ_t^{sp}) was decomposed using the benchmark debt tax ($\tau_t^{\star,sp}$) that a rational social planner would levy on rational households, as established in the literature. As shown in equation (48), the paternalistic social planner's optimal tax can be expressed as:

$$\tau_t^{sp} = \underbrace{\frac{E_t[u_T(t+1)]}{E_t^{\theta}[u_T(t+1)]}\tau_t^{\star,sp}}_{\substack{\text{PE-DE} \\ \text{interaction corrected}}} + \underbrace{\frac{E_t[u_T(t+1)] - E_t^{\theta}[u_T(t+1)]}{E_t^{\theta}[u_T(t+1)]}}_{\text{DE corrected}},$$

In this specification, the optimal debt tax is comprised of two distinct components: (i) the baseline tax from Bianchi (2011) targeting the pecuniary externality ($\tau_t^{\star,sp}$), which is adjusted by a slope coefficient capturing the discrepancy in marginal utilities between RE and DE (the PE-DE interaction-corrected term); and (ii) a correction for the differences in marginal utility growth rates between RE and DE (the DE-corrected term).

Figure 6 plots the original debt tax ($\tau_t^{\star,sp}$, black solid line), the PE-DE interaction-corrected term (red dashed line), and the total optimal debt tax under DE (τ_t^{sp}, blue dotted line). We find that the original debt tax moves in tandem with the PE-DE interaction-corrected term across the state space of bond holdings. In contrast, the DE-corrected term increases the optimal debt tax following favorable news and reduces it following negative news. In this sense, the DE-corrected term is the primary driver of the deviations between the optimal paternalistic tax and the standard macroprudential tax intended solely to address pecuniary externalities.

We simulate our baseline model under the paternalistic social planner and compute the long-run mean of the optimal tax, $\mathbb{E}[\tau_t^{sp}]$, using one million simulation periods. We find that the average optimal tax in our model is 2.07%, which is significantly lower than the 4.15% average tax ($\mathbb{E}[\tau_t^{\star,sp}]$) reported by Bianchi (2011). This result indicates that the policy component addressing diagnostic expectations (DE) offsets nearly half of the correction intended for pecu-

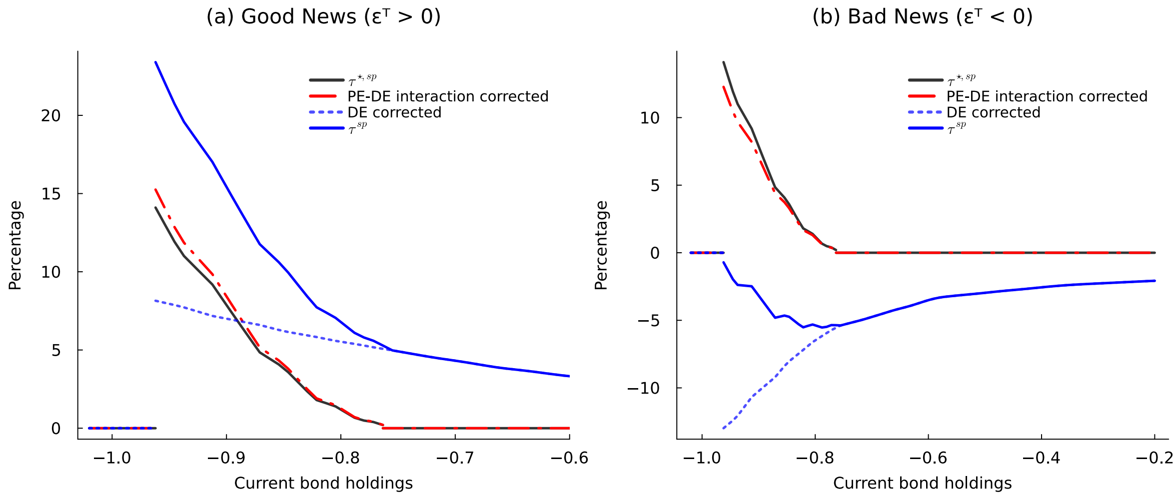

Figure 6: Optimal Debt Tax Decomposition

Notes: This figure shows the decomposed optimal debt tax implemented by the paternalistic (rational) social planner on the diagnostic private agent, when the current tradable income state is one standard deviation below the trend. $\tau^{\star,sp}$ (black solid line) represents the benchmark optimal debt tax for the rational social planner with rational households as we define in equation (47). τ^{sp} (blue solid line) represents the optimal debt tax of the paternalistic policymaker.

niary externalities.

Table 3 provides a further decomposition of the optimal tax. The pecuniary externality term, adjusted for its interaction with DE, remains the primary driver at 3.82 percentage points. Conversely, the pure DE-correction term contributes a substantial negative component of -1.75 percentage points. These findings highlight the importance of accounting for DE-driven credit cycles when calibrating optimal macroprudential policy. The DE-correction term is negative on average primarily because the paternalistic policymaker reduces the debt tax below -15 percent during crises—effectively providing a large subsidy—to counteract excessive pessimism and the resulting credit busts.

Table 3: Optimal Debt Tax Decomposition

$\mathbb{E}[\tau_t^{sp}]$	=	$\mathbb{E}$[PE-DE interaction corrected]	+	$\mathbb{E}$[DE corrected]
2.07%		3.82%		-1.75%

To better understand the interactions between the original optimal tax $\tau_t^{\star,sp}$ and the slope adjusted by DE, $Slope_t \equiv \frac{E_t[u_T(t+1)]}{E_t^{\theta}[u_T(t+1)]}$, we further decompose the first term of the optimal debt tax, $\mathbb{E}$[PE-DE interaction corrected] as follows:

$$\underbrace{\mathbb{E}[Slope_t \times \tau_t^{\star,sp}]}_{=3.82\%} = \overbrace{\underbrace{\mathbb{E}[Slope_t]}_{=0.98<1} \times \underbrace{\mathbb{E}[\tau_t^{\star,sp}]}_{=4.15\%}}^{=4.07\%} + \underbrace{\mathbb{COV}(Slope_t, \tau_t^{\star,sp})}_{=-0.25\%}.$$

On average, the difference in marginal utility between rational and diagnostic agents, $\mathbb{E}[Slope_t]$, reduces the baseline average tax ($\mathbb{E}[\tau_t^{\star,sp}]$) by 2 percent, resulting in a first term of 4.07 percentage points. Furthermore, the covariance between the slope and the tax, $\mathbb{COV}(Slope_t, \tau_t^{\star,sp})$, contributes an additional -0.25 percentage points. This negative covariance stems from two opposing effects following a positive transitory income news. First, under DE, over-optimism in response to good news induces a larger increase in consumption relative to RE, causing the slope term to rise. Second, the baseline debt tax ($\tau_t^{\star,sp}$) falls because, in high income states, the rational agents have weaker incentives to borrow and instead save to smooth future consumption, leading to lower debt levels and a diminished pecuniary externality relative to low income states. This offsetting mechanism explains why the total PE-DE interaction term (3.82 percent) is ultimately smaller than the original debt tax rate of 4.15 percent.

Table 4: Cyclicality of Optimal Debt Tax: Corr(Output, Tax)

	SP with RE	SP with DE
Households with RE	Procyclical (-0.76)	Procyclical (-0.89)
Households with DE	*Countercyclical (0.53)*	Procyclical (-0.82)

Notes: Correlations between the optimal debt tax and output are reported in parentheses. The table is structured by type of social planner (SP) in the columns and the type of households targeted by the policymaker's intervention in the rows. Both the social planner and the households are assumed to hold either RE or DE.

Table 4 presents the correlation between the optimal debt tax and output under four distinct behavioral configurations: (i) both the planner and households are rational; (ii) the planner is rational, while households are diagnostic; (iii) the planner is diagnostic, while households are rational; and (iv) both the planner and households are diagnostic.

The results demonstrate that the optimal debt tax is countercyclical only when a rational policymaker intervenes in a market of diagnostic households to achieve the rational social planner's allocation. In this case, the correlation between output and the optimal tax is 0.53, a result that remains robust across a wide range of diagnosticity parameters (see Table A.2). The other three configurations exhibit a procyclical debt tax, aligning with Schmitt-Grohé and Uribe (2017), who note that such procyclicality often contradicts conventional policy wisdom.

In the optimal tax formula (48), both the DE-correction term and the slope of the PE-DE interaction term vanish if the households and the social planner share identical expectations (i.e., both are rational or both share the same degree of diagnosticity). Notably, the DE-correction term is the primary driver of the countercyclical tax response when a rational planner faces diagnostic households. Conversely, when the social planner is diagnostic and households are rational, the sign of the DE-correction term flips, thereby reinforcing the procyclicality of the tax.

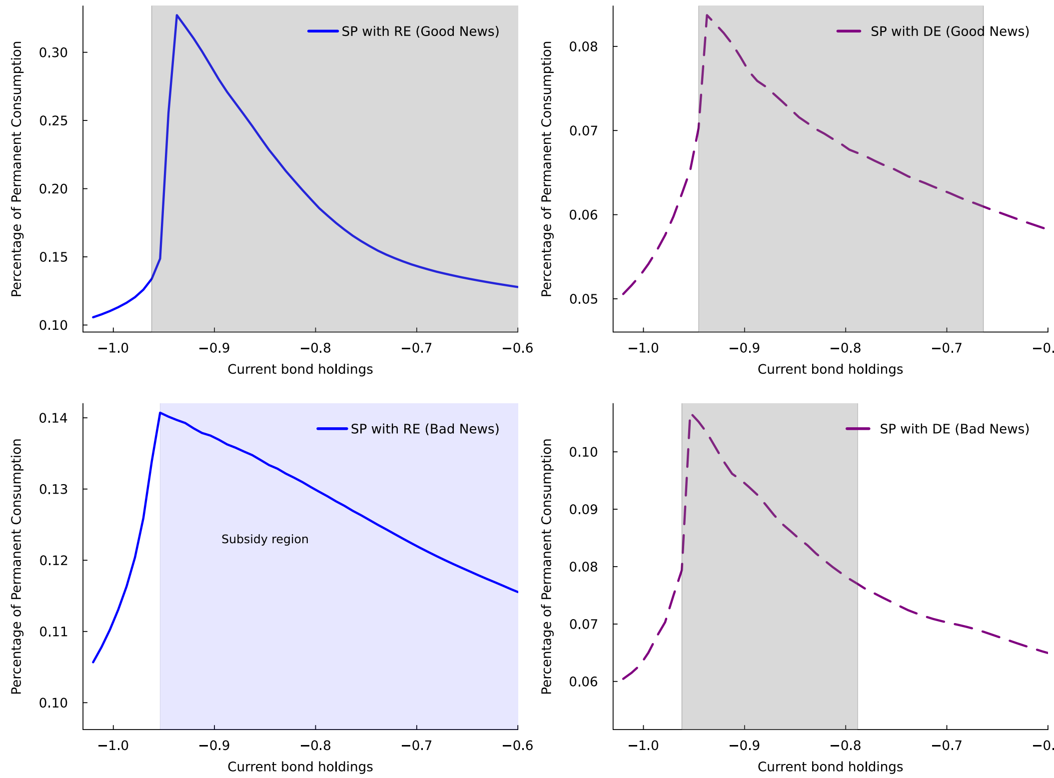

Figure 7: Welfare Gains

Notes: This figure illustrates the welfare gains achieved by correcting externalities under both the paternalistic planner (SP with RE) and the non-paternalistic planner (SP with DE), for a tradable income state one standard deviation below the trend. The gray-shaded area denotes the region where the optimal tax is positive, while the blue-shaded area represents the region where the optimal tax is negative, implying that the planner provides a subsidy to household borrowing.

We quantify welfare using consumption-equivalent welfare gains, defined as the constant percentage increase in consumption that makes an agent indifferent between the competitive equilibrium and the constrained-efficient allocation. Specifically, the welfare gain $W(s,b)$ at state (s,b) resulting from the social planner's intervention is given by:

$$\mathrm{V}^{ce}(s,b)\big(1+W(s,b)\big)^{(1-\gamma)} = \mathrm{V}^{sp}(s,b). \tag{49}$$

The welfare gains for each planner's allocation are evaluated under the expectations of the respective policymaker. In particular, when discussing the welfare gains associated with paternalistic policy, we compute the value functions under RE.

Figure 7 compares the welfare function $W(s,b)$ for paternalistic (left panel) and non-paternalistic (right panel) planners in low-income states. Paternalistic welfare gains increase with debt levels, as higher debt raises crisis probability. Consequently, policy benefits peak when the borrowing

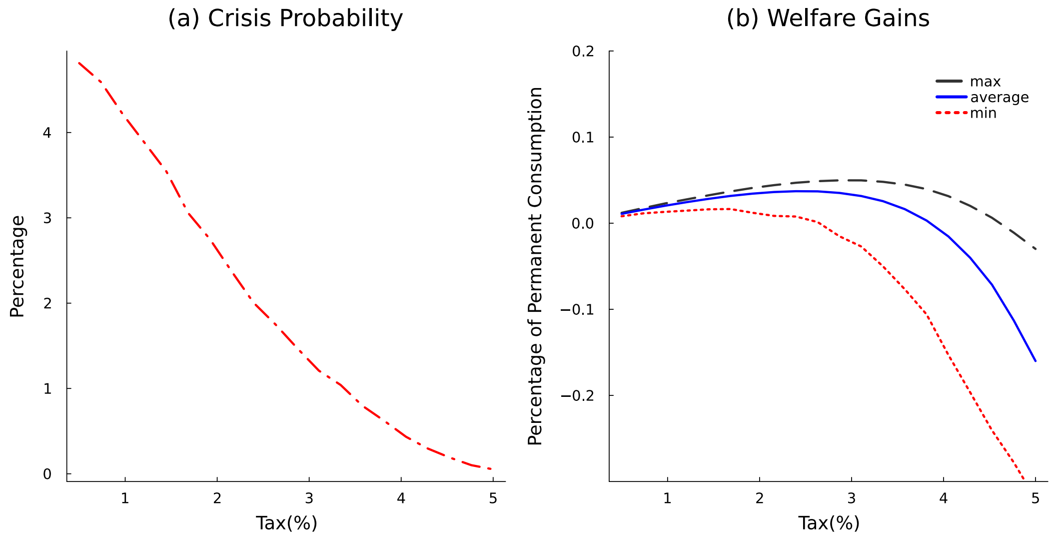

Figure 8: Effects of Fixed Taxes on Probability of Crises and Welfare
Notes: This figure shows the probability of crisis and the welfare gains as a function of the fixed tax. Welfare gains are computed as in equation (49) by replacing the value function of social planner with the competitive equilibrium which is adjusted by the fixed tax.

constraint is most likely to bind. Paternalistic gains significantly exceed non-paternalistic ones because the former rectifies both belief distortions and pecuniary externalities. This is particularly evident following good news—with maximum gains of 0.33% versus 0.08%—as the tax preempts crises triggered by sudden news reversals.

5.5 Paternalistic, Non-paternalistic and Simple Macroprudential Policies

In this section, we examine simple macroprudential policies that are feasible for real-world implementation and compare them against the optimal benchmarks, including both paternalistic and non-paternalistic policies. Following Bianchi and Mendoza (2018), we define the simple policy rule as follows:

$$\tau_t = \max\left[0, (1+\tau^*)(b_{t+1}/\bar{b})^{\chi} - 1\right]. \tag{50}$$

First, we determine the best fixed tax, τ^*, that maximizes welfare gains (as shown in Figure 8). In our simulations, this value is found to be 2.4%. Next, holding $\tau^* = 0.024$ constant, we search for the best reference debt level, $\bar{b}$, and sensitivity parameter, χ. Under DE, the welfare-maximizing combination is $\bar{b} = -0.84$ and $\chi = 0.2$.[16] These results suggest that the best simple tax schedule is significantly more acyclical under DE than under RE. This occurs because ad-

[16]Under rational expectations (RE), the optimal value of χ is 3.7, which is substantially larger and aligns with the results in Bianchi and Mendoza (2018).

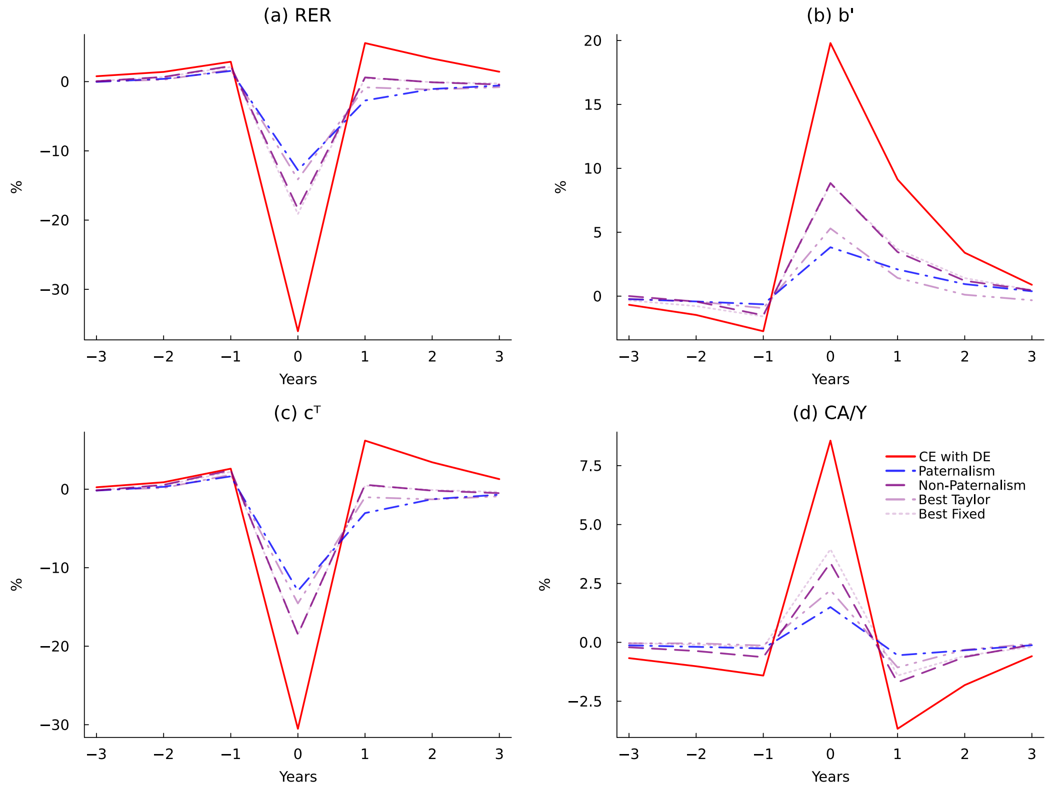

Figure 9: Crises Dynamics: Optimal Policy and Simple Policies

Notes: The panels (a), (b), and (c) are plotted as percent differences relative to the ergodic mean. Panel (d) shows HP detrended current-account to GDP ratio with a smoothing parameter 10.

justing the tax rate based solely on debt levels can induce significant welfare losses. Specifically, while the optimal policy dictates tax cuts in response to negative income news, simple, non-news-contingent rules fail to provide this necessary state-contingent relief. Consequently, as illustrated in Figure A.12, welfare gains decline substantially as the elasticity of the tax rate increases.

Figure 9 plots the average dynamics of aggregate variables around financial crises in the model simulation. As expected, the paternalistic social planner achieves the most significant cyclical smoothing, followed by the simple debt tax rule, the non-paternalistic planner, and the fixed tax.

Table 5 summarizes the welfare properties and crisis probabilities associated with different policy regimes. We find that the paternalistic planner achieves the highest welfare gain, representing a 0.15% increase in permanent consumption, while minimizing the consumption drop during crises to 4.63%. Importantly, our paternalistic framework reproduces a positive correlation between output and the debt tax. This result aligns with the conventional view of countercyclical macroprudential policy, as discussed in Schmitt-Grohé and Uribe (2017). In contrast,

Table 5: Performance of Optimal and Simple Policy Rules

	CE with DE	Paternalism	Non-Paternalism	Best Taylor	Best Fixed
Welfare Gains (%)	-	0.15	0.08	0.042	0.038
Crisis Probability (%)	5.53	1.20	0.99	0.87	1.86
Consumption Drop (%)	11.15	4.36	6.26	4.84	6.40
Tax Moments					
Mean of Tax (%)	-	2.07	4.12	4.06	2.40
Corr(Debt, Tax)	-	0.001	0.06	0.19	-
Corr(Output, Tax)	-	0.53	-0.82	0.85	-

Notes: Paternalism means the optimal policy that is implemented by the rational social planner to the diagnostic households. Non-Paternalism indicates the economy with the optimal policy that is implemented by the diagnostic social planner to the diagnostic households. "Mean of Tax" as presented under the Best Fixed column refers to the fixed tax rate that maximizes welfare.

the original Bianchi (2011) model produces a negative (procyclical) correlation. As shown in Table A.2, this positive correlation is robust to alternative parameterizations, provided that the diagnosticity parameter (θ) remains significantly above zero.

Figure 10 presents the average tax dynamics around the financial crises. To construct the figure, we first simulate one million paths of the tradable endowment, y^T. A financial crisis is then defined as a period in which the credit constraint is binding and net capital outflows exceed one standard deviation of their ergodic distribution mean in the competitive equilibrium under DE. For each simulated endowment path and the identified crisis dates, we calculate the corresponding optimal tax paths for both paternalistic and non-paternalistic planners. The resulting average paths are plotted in percentage. Panel (a) displays various debt tax rates, while Panel (b) shows the decomposed components of the paternalistic policymaker's tax, τ_t^{sp}.[17]

Our results demonstrate that by imposing a positive debt tax during the pre-crisis period, the planner effectively prevents crises that would otherwise occur in the competitive equilibrium. During a crisis, the paternalistic planner reduces the debt tax rate, primarily to counteract the sharp reversal in sentiment (over-pessimism).[18] This policy shift occurs because, during

[17] In our implementation, the macroprudential tax is set to zero whenever the social planners' borrowing constraint binds. Importantly, however, these binding episodes for the social planner do not necessarily coincide with crisis periods as defined by private agents under DE. At the crisis date (year 0), the probability that the planner's borrowing constraint binds is only 21.65% for the rational social planner and 17.96% for the diagnostic social planner. Consequently, as shown in Figure 10, the optimal tax in both the paternalistic and non-paternalistic cases does not mechanically drop to zero during crisis episodes. This is because policymakers often succeed in avoiding a binding borrowing constraint by implementing the optimal taxes ex-ante, thereby preempting the crisis that would have otherwise occurred in a competitive equilibrium without taxes.

[18] The social planner is constrained to respect the borrowing constraint at all times. By subsidizing debt to address pessimism during crises, the planner does not circumvent the constraint; rather, these subsidies boost tradable consumption and the real exchange rate, thereby effectively relaxing the borrowing constraint during crisis

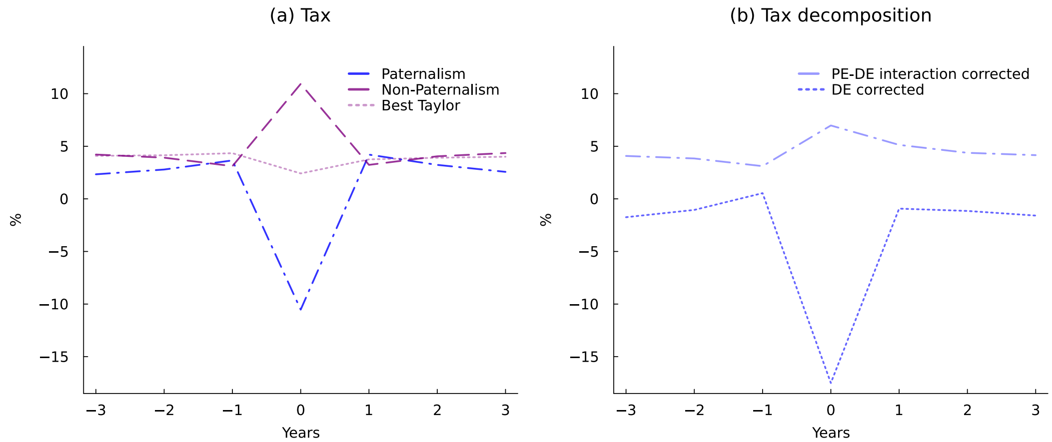

Figure 10: Tax Dynamics around Financial Crises

Notes: This figure shows the average dynamics of various tax policies around Sudden Stop events, based on one million simulations of the tradable endowment, y^T. A crisis is defined as a period where the credit constraint binds and capital outflows exceed one standard deviation of their ergodic mean in the competitive equilibrium under diagnostic households. For each simulation, we calculate the optimal tax paths for both paternalistic and non-paternalistic planners. Panel (a) displays the average paths of various debt taxes, while Panel (b) decomposes the paternalistic planner's tax, τ_t^{sp}.

such periods, the correction for belief distortions outweighs the correction for the pecuniary externality. This dominance results in a countercyclical optimal tax, which stands in sharp contrast to the procyclical tax implemented by the non-paternalistic planner.[19]

In contrast, the tax schedule under the simple Taylor-type rule exhibits much more attenuated adjustments. This finding helps explain the observation by Fernández et al. (2015) that capital controls often vary little in practice. The intuition is that it is often preferable to maintain a stable policy if that policy cannot be fully state-contingent. The simple debt rule (50) is not contingent on the news state (ε^T) and depends solely on credit cycles ($\frac{b_{t+1}}{\bar{b}}$). This creates a fundamental conflict: debt may be increasing (signaling a tax hike) at the same time that negative news is realized (signaling a tax cut to alleviate over-pessimism). In such instances, adjusting the tax based only on the credit cycle can induce significant welfare losses.

periods.

[19]Figure A.13 illustrates that output and debt levels collapse during typical crises. The combined dynamics of the debt tax and output reveal that the optimal tax is countercyclical under the paternalistic planner but procyclical under the non-paternalistic planner. Notably, the reduction in debt is more pronounced under the non-paternalistic regime, partly because the debt tax rises in that case while it falls under the paternalistic planner during crises.

6 Conclusion

In this paper, we integrate DE into the traditional small open economy model with an occasionally binding constraint. Our findings suggest that DE can intensify crisis dynamics and produce realistic boom-bust credit cycles through a combination of overreaction to new information, risk neglect, and borrowing constraints. Our model includes scenarios of both overborrowing and underborrowing as equilibrium outcomes. In contrast, the standard model suggests cutting the optimal tax on debt during prosperous times, whereas policymakers advocate for increasing debt taxes to mitigate economic booms. Our model aligns with the countercyclical optimal debt taxes, reconciling the standard quantitative model with policymakers' conventional wisdom.

Future avenues for research will be to incorporate more ingredients or modify features in the model to better explain the stylized facts. One way is to modify the belief updating system such that agents have distant memory when perceiving news, as shown in Bianchi et al. (2024a). This modification would generate more realistic credit boom-bust cycles in which bust is followed by slow recoveries as shown in the empirical literature on financial crises (e.g., Reinhart and Rogoff (2014)). Another way is to incorporate financial intermediaries into the small open economy model with DE. As most macroprudential policy and capital controls are imposed on financial intermediaries in practice, for example in the form of capital ratios guided by the Bank for International Settlement (BIS), the enriched model would deliver further practical policy implications.

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Appendix

A 3-period Model Proofs

A.1 Proof of Proposition 1

Let μ_2 denote the Lagrangian multiplier for the borrowing constraint $b_2 \geq -\kappa y_2$. The first-order conditions are given by

$$u'(c_1) = E_1[u'(c_2)], \tag{A.1}$$

$$E_1[u'(c_2)] = E_1[u'(c_3)] + \mu_2, \tag{A.2}$$

where the marginal utility of consumption is $u'(c) = \delta - \gamma c$.

Suppose the borrowing constraint does not bind ($\mu_2 = 0$). Then we can use the following first-order conditions

$$c_1 = E_1[c_2], \tag{A.3}$$

$$E_1[c_2] = E_1[c_3]. \tag{A.4}$$

We proceed with the method of undetermined coefficients. Conjecture that the policy functions for savings b_1 and b_2 as follows:

$$b_1^* = \alpha_{b,1}^{RE} b_0 + \alpha_{y,1}^{RE} y_1, \tag{A.5}$$

$$b_2^* = \alpha_{b,2}^{RE} b_1 + \alpha_{y,2}^{RE} y_2. \tag{A.6}$$

It is optimal not to save in the final period, so we have $b_3^* = 0$. Next, we plug the conjectured solution b_2^* into equation (A.4) with relevant budget constraints. Then we find that $\alpha_{b,2}^{RE} = \frac{1}{2}$ and $\alpha_{y,2}^{RE} = \frac{1}{2}(1-\rho)$.

Given b_2^*, plug the conjectured solution b_1^* into equation (A.3) and we get $\alpha_{b,1}^{RE} = \frac{2}{3}$ and $\alpha_{y,1}^{RE} = \frac{2}{3}\left(1 - \frac{\rho(1+\rho)}{2}\right)$.

If the borrowing constraint binds ($\mu_2 > 0$), we have

$$b_2^* = -\kappa y_2.$$

Given b_2^*, plug the conjectured solution b_1^* into equation (A.3) and get

$$b_1^* = \frac{1}{2}b_0 + \frac{1}{2}[1-(1+\kappa)\rho]y_1.$$

The borrowing constraint binds if and only if $b_2^* < -\kappa y_2$. Plug the desired saving b_2^* when the constraint is slack and use $y_2 = \rho y_1 + \varepsilon_2$ and rearrange in terms of ε_2. Then, we have

$$\varepsilon_2 < \bar{\varepsilon}_2^{RE} \equiv -\frac{b_1 + (1-\rho+2\kappa)\rho y_1}{1-\rho+2\kappa}.$$

In this environment, there are no externalities or distortions that hinder the agents in the competitive equilibrium from choosing the constrained social planner's allocation. Thus, the competitive equilibrium is Pareto optimum as implied by the First Welfare Theorem.

A.2 Proof of Proposition 2

Let μ_2^θ denote the Lagrangian multiplier for the borrowing constraint $b_2^\theta \geq -\kappa y_2$ when the agent forms DE. For notational simplicity, we drop superscript θ for variables y, c, and b. Then, first-order conditions with DE agents are given by

$$c_1 = E_1^\theta[c_2], \tag{A.7}$$

$$E_1^\theta[c_2] = E_1^\theta[c_3] + \mu_2^\theta. \tag{A.8}$$

Suppose the borrowing constraint does not bind ($\mu_2^\theta = 0$). Then we can use the following first-order conditions:

$$c_1 = E_1^\theta[c_2], \tag{A.9}$$

$$E_1^\theta[c_2] = E_1^\theta[c_3]. \tag{A.10}$$

Analogous to the RE case, conjecture that decision rules for savings b_1^θ and b_2^θ are given by

$$b_1^{\theta*} = \alpha_{b,1}^\theta b_0 + \alpha_{y,1}^\theta y_1 + \alpha_{\varepsilon,1}^\theta \varepsilon_1, \tag{A.11}$$

$$b_2^{\theta*} = \alpha_{b,2}^\theta b_1 + \alpha_{y,2}^\theta y_2 + \alpha_{\varepsilon,2}^\theta \varepsilon_2. \tag{A.12}$$

By applying the DE equation (12) on y_2, we can verify that

$$E_1^\theta[y_2] = E_1[y_2] + \theta\big(E_1[y_2] - E_0[y_2]\big) = \rho y_1 + \theta\rho\varepsilon_1. \tag{A.13}$$

Under the assumption of DE on exogenous variables, we do not apply the DE operator on endogenous variables. Since previous saving b_0 and income y_1 are known at time 1, saving at time 1 b_1, defined in (A.11), is known at time 1 as well:

$$E_1^\theta[b_1^\theta] = E^\theta[b_1^\theta | b_0, y_1, \varepsilon_1] = b_1. \tag{A.14}$$

Similarly, we have

$$E_2^\theta[b_2^\theta] = E^\theta[b_2^\theta | b_1, y_2, \varepsilon_2] = b_2. \tag{A.15}$$

We solve the problem using backward induction. At time 2, the agents' first-order condition is given by

$$c_2 = E_2^\theta[c_3]. \tag{A.16}$$

We know that $b_3^{\theta *} = 0$. Equation (A.16) can be rewritten as

$$y_2 + b_1^\theta - b_2^\theta = E_2^\theta[y_3 + b_2^\theta] = \rho y_2 + \theta \rho \varepsilon_2 + b_2^\theta. \tag{A.17}$$

Rearranging equation (A.17) provides the solution for b_2^θ as follows:

$$b_2^{\theta *} = \frac{1}{2} b_1 + \frac{1}{2}(1-\rho) y_2 - \frac{1}{2} \rho \theta \varepsilon_2. \tag{A.18}$$

We can verify that $b_2^{\theta *}$ satisfies the original first-order condition (A.10), and thus the solution is time-consistent.

Rewrite (A.9) using the relevant budget constraints:

$$y_1 + b_0 - b_1 = E_1^\theta[y_2 + b_1 - b_2] = \rho y_1 + \theta \rho \varepsilon_1 + b_1 - E_1^\theta[b_2^{\theta *}]. \tag{A.19}$$

Given the solution $b_2^{\theta *}$ and (A.13), we have

$$E_1^\theta[b_2^{\theta *}] = E_1^\theta[\frac{1}{2} b_1 + \frac{1}{2}(1-\rho) y_2 - \frac{1}{2} \rho \theta \varepsilon_2] = \frac{1}{2} b_1 + \frac{1}{2}(1-\rho)(\rho y_1 + \theta \rho \varepsilon_1). \tag{A.20}$$

where ε_2 is perceived as *i.i.d.* given y_1 and ε_1, implying $E_1^\theta[\varepsilon_2] = 0$.

Combining the above two equations give us the solution for b_1^θ as follows:

$$b_1^{\theta *} = \frac{2}{3} b_0 + \frac{2}{3}\left(1 - \frac{\rho(1+\rho)}{2}\right) y_1 - \frac{1}{3} \rho (1+\rho) \theta \varepsilon_1. \tag{A.21}$$

If the borrowing constraint binds ($\mu_2^\theta > 0$), we have

$$b_2^{\theta *} = -\kappa y_2.$$

To define the crisis probability, note that the borrowing constraint binds if and only if $b_2^{\theta*} < -\kappa y_2$. Plug the desired saving $b_2^{\theta*}$ when the constraint is slack and use $y_2 = \rho y_1 + \varepsilon_2$ and rearrange in terms of ε_2. Then, we have

$$\left[\rho\theta - (1-\rho+2\kappa)\right]\varepsilon_2 > b_1 + (1-\rho+2\kappa)\rho y_1.$$

If $\rho\theta - (1-\rho+2\kappa) < 0$, we have

$$\varepsilon_2 < \bar{\varepsilon}_2^{\theta} \equiv \frac{b_1 + (1-\rho+2\kappa)\rho y_1}{\rho\theta - (1-\rho+2\kappa)} < 0, \tag{A.22}$$

and the probability of crises is given by

$$\Pr(\varepsilon_2 < \bar{\varepsilon}_2^{\theta}) = F(\bar{\varepsilon}_2^{\theta}). \tag{A.23}$$

Moreover, we have

$$\frac{\partial \bar{\varepsilon}_2^{\theta}}{\partial \theta} < 0. \tag{A.24}$$

Thus, the probability of crises decreases in θ:

$$\frac{\partial \Pr(\varepsilon_2 < \bar{\varepsilon}_2^{\theta})}{\partial \theta} < 0.$$

Meanwhile, if $\rho\theta - (1-\rho+2\kappa) > 0$, the above equation is rearranged as

$$\varepsilon_2 > \frac{b_1 + (1-\rho+2\kappa)\rho y_1}{\rho\theta - (1-\rho+2\kappa)} = \bar{\varepsilon}_2^{\theta} > 0, \tag{A.25}$$

and the probability of crises is given by

$$\Pr(\varepsilon_2 > \bar{\varepsilon}_2^{\theta}) = 1 - F(\bar{\varepsilon}_2^{\theta}), \tag{A.26}$$

where the probability of crises increases in θ due to the following result:

$$\frac{\partial \bar{\varepsilon}_2^{\theta}}{\partial \theta} < 0. \tag{A.27}$$

A.3 Proof of Proposition 3

Again, we conjecture the solutions as follows:

$$b_1^{\theta*} = \alpha_{b,1}^{\theta} b_0 + \alpha_{y,1}^{\theta} y_1 + \alpha_{\varepsilon,1}^{\theta} \varepsilon_1, \tag{A.28}$$

$$b_2^{\theta *} = \alpha_{b,2}^{\theta} b_1 + \alpha_{y,2}^{\theta} y_2 + \alpha_{\varepsilon,2}^{\theta} \varepsilon_2. \tag{A.29}$$

If the borrowing constraint does not bind, first-order conditions are given by

$$c_1 = E_1^{\theta}[c_2], \tag{A.30}$$
$$E_1^{\theta}[c_2] = E_1^{\theta}[c_3]. \tag{A.31}$$

Rewriting (A.31) and using $b_3 = 0$, we get

$$E_1^{\theta}[y_2 + b_1 - b_2] = E_1^{\theta}[y_3 + b_2]. \tag{A.32}$$

By applying the DE operator (17) to the above equation, we have

$$\begin{aligned} & E_1[y_2 + b_1 - b_2] + \theta\big(E_1[y_2 + b_1 - b_2] - E_0[y_2 + b_1 - b_2]\big) \\ & = E_1[y_3 + b_2] + \theta\big(E_1[y_3 + b_2] - E_0[y_3 + b_2]\big). \end{aligned} \tag{A.33}$$

Using (5), (A.28) and (A.29), we have

$$E_1[y_2] + \theta\big(E_1[y_2] - E_0[y_2]\big) = \rho y_1 + \theta \rho \varepsilon_1, \tag{A.34}$$
$$E_1[y_3] + \theta\big(E_1[y_3] - E_0[y_3]\big) = \rho^2 y_1 + \theta \rho^2 \varepsilon_1, \tag{A.35}$$
$$E_1[b_1] + \theta\big(E_1[b_1] - E_0[b_1]\big) = b_1 + \theta(\alpha_{y,1}^{\theta} + \alpha_{\varepsilon,1}^{\theta})\varepsilon_1, \tag{A.36}$$
$$E_1[b_2] + \theta\big(E_1[b_2] - E_0[b_2]\big) = \alpha_{b,2}^{\theta} b_1 + \alpha_{y,2}^{\theta} \rho y_1 + \theta\big(\alpha_{b,2}^{\theta}(\alpha_{y,1}^{\theta} + \alpha_{\varepsilon,1}^{\theta}) + \alpha_{y,2}^{\theta}\rho\big)\varepsilon_1. \tag{A.37}$$

Since the RE is a linear operator, we can plug the above four equations into (A.33) and match coefficients to get

$$\alpha_{b,2}^{\theta} = \frac{1}{2}, \qquad \alpha_{y,2}^{\theta} = \frac{1}{2}(1-\rho). \tag{A.38}$$

Rewrite (A.30) using the relevant budget constraints, and we have

$$y_1 + b_0 - b_1 = E_1^{\theta}[y_2 + b_1 - b_2], \tag{A.39}$$

and applying the DE operator (17) to the above equation gives

$$y_1 + b_0 - b_1 = E_1[y_2 + b_1 - b_2] + \theta\big(E_1[y_2 + b_1 - b_2] - E_0[y_2 + b_1 - b_2]\big). \tag{A.40}$$

Plug (A.28), (A.34), (A.36), (A.37) into (A.40) and match coefficients to get

$$\alpha^{\theta}_{b,1} = \frac{2}{3}, \qquad \alpha^{\theta}_{y,1} = \frac{2}{3}\left(1 - \frac{\rho(1+\rho)}{2}\right), \qquad \alpha^{\theta}_{\varepsilon,1} = -\frac{2\theta}{3(3+\theta)}(1+\rho(1+\rho))\varepsilon_1. \tag{A.41}$$

At time 2, the agent's first-order condition is given by

$$c_2 = E^{\theta}_2[c_3], \tag{A.42}$$

or equivalently

$$y_2 + b_1 - b_2 = E^{\theta}_2[y_3 + b_2] = E_2[y_3 + b_2] + \theta\big(E_2[y_3 + b_2] - E_1[y_3 + b_2]\big), \tag{A.43}$$

where we apply the DE operator (17).

Using (5) and (A.29), we have

$$E_2[y_3] + \theta\big(E_2[y_3] - E_1[y_3]\big) = \rho y_2 + \theta\rho\varepsilon_2, \tag{A.44}$$

$$E_2[b_2] + \theta\big(E_2[b_2] - E_1[b_2]\big) = b_2 + \theta(\alpha_{y,2} + \alpha_{\varepsilon,2})\varepsilon_2. \tag{A.45}$$

Plug the above equations into (A.43) with (A.29) and match coefficients to get

$$\alpha^{\theta}_{\varepsilon,2} = -\frac{\theta}{2(2+\theta)}(1+\rho)\varepsilon_2. \tag{A.46}$$

The solution for $\tilde{b}^{\theta}_2$ (19) satisfies the original first-order condition (A.31), implying that solutions are time-consistent. This time-consistency result is analogous to Bianchi et al. (2024a).

If the borrowing constraint binds, we combine the first-order condition at time 1 (A.30) and the binding borrowing constraint, $\tilde{b}^{\theta}_2 = -\kappa y_2$. Then, we have

$$y_1 + b_0 - b_1 = E^{\theta}_1[y_2 + b_1 - b_2] = E^{\theta}_1[(1+\kappa)y_2 + b_1]. \tag{A.47}$$

Plug the conjectured solution for $\tilde{b}^{\theta}_1$ (A.28) into the above equation and match coefficients:

$$\tilde{\alpha}^{\theta}_{b,1} = \frac{1}{2}, \qquad \tilde{\alpha}^{\theta}_{y,1} = \frac{1-(1+\kappa)\rho}{2}, \qquad \tilde{\alpha}^{\theta}_{\varepsilon,1} = -\frac{\theta}{2(2+\theta)}[1+(1+\kappa)\rho]. \tag{A.48}$$

The proof for the crisis probability is analogous to that under DE on exogenous variables.

B Proofs

B.1 Recursive Formation

We show that the sequential and recursive problems are identical under DE on *exogenous* variables.

Under the assumption of DE on exogenous variables, the DE operator on function h at time $t+1$ conditional on time t information is defined as follows:

$$E_t^\theta[h_{t+1}] = E_t^\theta[h(y_{t+1}^T, \varepsilon_{t+1}^T)] \equiv \int h(y_{t+1}^T, \varepsilon_{t+1}^T(y_{t+1}^T, y_t^T)) f^\theta(y_{t+1}^T | y_t^T, \varepsilon_t^T) dy_{t+1}^T. \tag{A.49}$$

The function $h(X)$ is assumed to be Borel measurable and integrable, i.e., $E^\theta[|h(X)|] < \infty$, to ensure that the (conditional) expectation $E^\theta[h(X)]$ is well-defined and finite. $f^\theta(y_{t+1}^T | y_t^T, \varepsilon_t^T)$ is diagnostically distorted probability density function of y_{t+1}^T conditional on the exogenous state variables y_t^T and ε_t^T. News at time $t+1$ (ε_{t+1}^T) is a function of y_{t+1}^T and y_t^T, which is implied by the true data generating process, $\log y_{t+1}^T = \rho_T \log y_t^T + \varepsilon_{t+1}^T$.

Start from the sequential problem. Let $\Gamma(y_t^T, b_t)$ denote the feasible set of choice variables (c_t and b_{t+1}) that satisfy the budget constraint (29) and the borrowing constraint (30) at time t, given y_t^T and b_t. The value function at time t then satisfies

$$V_t(y_t^T, \varepsilon_t^T, b_t) = \max_{\{c_s, b_{s+1}\}_{s=t}^\infty} u(c_t) + E_t^\theta\left[\sum_{s=t+1}^\infty \beta^{s-t} u(c_s)\right] \text{ s.t. } \{c_s, b_{s+1}\} \in \Gamma(y_s^T, b_s),\ \forall s \geq t \text{ and } b_t \text{ given.} \tag{A.50}$$

Exploiting the additive separability of preferences and the definition of the DE operator (A.49), we can alternatively represent this condition recursively:

$$\begin{aligned}
&V_t(y_t^T, \varepsilon_t^T, b_t) = \max_{\{c_s, b_{s+1}\}_{s=t}^\infty} u(c_t) + E_t^\theta\left[\sum_{s=t+1}^\infty \beta^{s-t} u(c_s)\right] \text{ s.t. } \{c_s, b_{s+1}\} \in \Gamma(y_s^T, b_s),\ \forall s \geq t \text{ and } b_t \text{ given.} \\
&= \max_{c_t, b_{t+1}} \left(u(c_t) + \max_{\{c_s, b_{s+1}\}_{s=t+1}^\infty} E_t^\theta\left[\sum_{s=t+1}^\infty \beta^{s-t} u(c_s)\right] \text{ s.t. } \{c_s, b_{s+1}\} \in \Gamma(y_s^T, b_s),\ \forall s \geq t+1,\ b_{t+1} \text{ given.}\right) \\
&\quad \text{s.t. } \{c_t, b_{t+1}\} \in \Gamma(y_t^T, b_t),\ b_t \text{ given.} \\
&= \max_{c_t, b_{t+1}} \left(u(c_t) + \beta \max_{\{c_s, b_{s+1}\}_{s=t+1}^\infty} E_t^\theta\left[E_{t+1}^\theta\left[\sum_{s=t+1}^\infty \beta^{s-(t+1)} u(c_s)\right]\right] \text{ s.t. } \{c_s, b_{s+1}\} \in \Gamma(y_s^T, b_s),\ \forall s \geq t+1,\ b_{t+1} \text{ given.}\right) \\
&\quad \text{s.t. } \{c_t, b_{t+1}\} \in \Gamma(y_t^T, b_t),\ b_t \text{ given.} \\
&= \max_{c_t, b_{t+1}} \left(u(c_t) + \beta \max_{\{c_s, b_{s+1}\}_{s=t+1}^\infty} E_t^\theta\left[E_{t+1}^\theta[u(c_{t+1})] + E_{t+1}^\theta\left[\sum_{s=t+2}^\infty \beta^{s-(t+1)} u(c_s)\right]\right] \text{ s.t. } \{c_s, b_{s+1}\} \in \Gamma(y_s^T, b_s),\ \forall s \geq t+1,\ b_{t+1} \text{ given.}\right) \\
&\quad \text{s.t. } \{c_t, b_{t+1}\} \in \Gamma(y_t^T, b_t),\ b_t \text{ given.}
\end{aligned}$$

$$= \max_{c_t, b_{t+1}} \left(u(c_t) + \beta E_t^\theta \left[\max_{\{c_s, b_{s+1}\}_{s=t+1}^\infty} u(c_{t+1}) + E_{t+1}^\theta \left[\sum_{s=t+2}^\infty \beta^{s-(t+1)} u(c_s) \right] \text{ s.t. } \{c_s, b_{s+1}\} \in \Gamma(y_s^T, b_s),\ \forall s \geq t+1,\ b_{t+1} \text{ given.} \right] \right)$$

$$\text{s.t. } \{c_t, b_{t+1}\} \in \Gamma(y_t^T, b_t),\ b_t \text{ given.}$$

$$= \max_{c_t, b_{t+1}} u(c_t) + \beta E_t^\theta \left[V_{t+1}(y_{t+1}^T, \varepsilon_{t+1}^T, b_{t+1}) \right] \text{ s.t. } \{c_t, b_{t+1}\} \in \Gamma(y_t^T, b_t),\ b_t \text{ given.}$$

The law of iterated expectations has been proved to hold under DE on endogenous variables for immediate memory ($J = 1$), as shown in Lemma 1 of Bianchi et al. (2024a). In the third line of our derivation, we exploit similar property. Given the definition of the DE operator (A.49) (under DE on exogenous variables), the law of iterated expectations holds as follows:

$$E_t^\theta E_{t+1}^\theta [h_{t+1+n}] = E_t^\theta [h_{t+1+n}]. \tag{A.51}$$

The fourth line applies the additivity property of the DE operator for any functions h and g.

$$E_t^\theta [h_{t+n} + g_{t+n}] = E_t^\theta [h_{t+n}] + E_t^\theta [g_{t+n}]. \tag{A.52}$$

The fifth line again applies the law of iterated expectations: $E_t^\theta E_{t+1}^\theta [u(c_{t+1})] = E_t^\theta [u(c_{t+1})]$. The final line replaces the recursive expression with the definition of the value function from (A.50).

B.2 Proof of Proposition 4

Let $\tau_t^{\theta,sp}$ be the state contingent tax charged on debt issued at time t by the non-paternalistic policymaker. Then, the Euler equation for the regulated economy under DE becomes

$$u_T(t) = \beta(1+r)(1+\tau_t^{\theta,sp}) E_t^\theta [u_T(t+1)] + \mu_t. \tag{A.53}$$

Note that the debt tax is rebated as a lump sum transfer $T_t = b_t(1+r)\tau_{t-1}^{\theta,sp}$.

Combining the optimality conditions for the diagnostic social planner (38) and (39), we have the following Euler equation for the diagnostic social planner:

$$u_T(t) = \beta(1+r) E_t^\theta \left[u_T(t+1) + \mu_{t+1}^{\theta,sp} \psi_{t+1}^\theta \right] + \mu_t^{\theta,sp}(1 - \psi_t^\theta). \tag{A.54}$$

The constrained-efficient allocations of the diagnostic social planner consist of a set of sequences $\{c_t^T, c_t^N, b_{t+1}, p_t^N, \mu_t^{\theta,sp}\}_{t\geq 0}$ that satisfies the following conditions: equation (33), market clearing conditions in definition 1, (40), (A.54), and $\mu_t^{\theta,sp} \geq 0$.

In contrast, the diagnostic decentralized equilibrium with the debt tax imposed by the non-paternalistic policymaker is characterized by $\{c_t^T, c_t^N, b_{t+1}, p_t^N, \mu_t, \tau_t^{\theta,sp}, T_t\}_{t\geq 0}$ that satisfies the following conditions: (33), (35), market clearing conditions in definition 1, (A.53), and $\mu_t \geq 0$.

By setting the debt tax as $\tau_t^{\theta,sp} = \frac{E_t^\theta\left[\mu_{t+1}^{\theta,sp}\psi_{t+1}^\theta\right]}{E_t^\theta[u_T(t+1)]}$, the constrained-efficient allocations of the diagnostic social planner are equivalent to those of the regulated economy under DE in states where $\mu_t^{\theta,sp} = 0$.

B.3 Proof of Proposition 5

Let τ_t^{sp} denote the state-contingent tax on debt issued at time t by the paternalistic policymaker. The subsequent steps proceed analogously to Proposition 4. The Euler equation for the regulated economy under DE, where the rational social planner imposes the debt tax, is given by

$$u_T(t) = \beta(1+r)(1+\tau_t^{sp})E_t^\theta\left[u_T(t+1)\right] + \mu_t. \tag{A.55}$$

Note that the debt tax is rebated as a lump sum transfer $T_t = b_t(1+r)\tau_{t-1}^{sp}$ and the Euler equation for the rational social planner is given by equation (44).

The constrained-efficient allocations of the rational social planner consist of a set of sequences $\{c_t^T, c_t^N, b_{t+1}, p_t^N, \mu_t^{sp}\}_{t\geq 0}$ that satisfies the following conditions: (33), market clearing conditions in Definition 1, (43), (44), and $\mu_t^{sp} \geq 0$.

On the other hand, the diagnostic decentralized equilibrium with the debt tax imposed by the paternalistic policymaker is characterized by a set of sequences $\{c_t^T, c_t^N, b_{t+1}, p_t^N, \mu_t, \tau_t^{sp}, T_t\}_{t\geq 0}$ that satisfies the following conditions: (33), (35), market clearing conditions in definition 1, (A.55), and $\mu_t \geq 0$.

By setting the debt tax as $\tau_t^{sp} = \frac{E_t\left[u_T(t+1) + \mu_{t+1}^{sp}\psi_{t+1}\right]}{E_t^\theta[u_T(t+1)]} - 1$, the constrained-efficient allocations for the rational social planner are equivalent to those of the regulated economy under DE in states where $\mu_t^{sp} = 0$.

B.4 Proof of Proposition 6

The optimal tax schedule of the paternalistic policymaker (46) can be rewritten as

$$\tau_t^{sp} = \frac{E_t\left[\mu_{t+1}^{sp}\psi_{t+1}\right]}{E_t^\theta\left[u_T(t+1)\right]} + \frac{E_t\left[u_T(t+1)\right]}{E_t^\theta\left[u_T(t+1)\right]} - \frac{E_t^\theta\left[u_T(t+1)\right]}{E_t^\theta\left[u_T(t+1)\right]}. \tag{A.56}$$

By multiplying $E_t\left[u_T(t+1)\right]$ to both the numerator and denominator of the first term on the right-hand side, we obtain

$$\tau_t^{sp} = \frac{E_t\left[u_T(t+1)\right]}{E_t^\theta\left[u_T(t+1)\right]} \times \frac{E_t\left[\mu_{t+1}^{sp}\psi_{t+1}\right]}{E_t\left[u_T(t+1)\right]} + \frac{E_t\left[u_T(t+1)\right]}{E_t^\theta\left[u_T(t+1)\right]} - \frac{E_t^\theta\left[u_T(t+1)\right]}{E_t^\theta\left[u_T(t+1)\right]}. \tag{A.57}$$

Since both optimal tax schedules (46) and (47) are evaluated at the constrained-efficient allocations of the rational social planner, the Lagrange multipliers μ_{t+1}^{sp} in the two equations coincide. Therefore, substituting the tax schedule (47) into equation (A.57) yields

$$\tau_t^{sp} = \frac{E_t\left[u_T(t+1)\right]}{E_t^{\theta}\left[u_T(t+1)\right]}\tau_t^{\star,sp} + \frac{E_t\left[u_T(t+1)\right] - E_t^{\theta}\left[u_T(t+1)\right]}{E_t^{\theta}\left[u_T(t+1)\right]}, \tag{A.58}$$

which is the desired result.

Note that the social planner's Lagrangian multiplier, μ_{t+1}^{sp}, coincides with the multiplier from Bianchi (2011)'s framework, $\mu_{t+1}^{\star,sp}$. The reason is that the allocations chosen by the rational social planner with diagnostic households correspond to the constrained Pareto optimum in Bianchi (2011).

C Production Economy

We extend our baseline endowment economy model to the two-sector production economy following Arce et al. (2025). To study the properties of optimal debt tax policy in isolation, we focus on the case in which the social planner controls only the level of external debt, taking labor and goods market clearing conditions as given.

Household's problem In each period, households are endowed with a fixed amount of labor, $\bar{h}$, which is perfectly mobile across the tradable and non-tradable sectors. Households supply labor inelastically, as they do not value utility from leisure. They receive competitive wages and firm profits, and they also choose consumption and bond holdings subject to a budget constraint and a credit constraint. The household budget constraint is given by

$$b_{t+1} + c_t^T + p_t^N c_t^N = w_t \bar{h} + \pi_t^T + \pi_t^N + (1+r) b_t, \tag{A.59}$$

where w_t denotes the wage, and π_t^T and π_t^N are profits from firms producing tradable and non-tradable goods, respectively. The borrowing constraint is given by

$$b_{t+1} \geq -\kappa (w_t \bar{h} + \pi_t^T + \pi_t^N). \tag{A.60}$$

Firm's problem The tradable and non-tradable goods are produced by competitive firms. Production technologies are given by

$$\max_{h_t^T} z_t^T (h_t^T)^\alpha - w_t h_t^T, \tag{A.61}$$

$$\max_{h_t^N} p_t^N z^N (h_t^N)^\alpha - w_t h_t^N, \tag{A.62}$$

where z_t^T is a stochastic productivity process in the tradable sector, z^N is a constant productivity level in the non-tradable sector, and $\alpha \in (0,1)$. We assume that the log of tradable-sector productivity, $\log z_t^T$, follows a first-order Markov process: $\log z_t^T = \rho_z \log z_{t-1}^T + \varepsilon_t^z$ where $\rho_z \in (0,1)$ denotes the persistence of productivity and ε_t^z is a Gaussian innovation with mean zero and standard deviation σ_z. As in the baseline endowment economy, diagnostic agents form distorted beliefs about future productivity. In particular, conditional on the current productivity level and the realized innovation, $(\log z_t^T, \varepsilon_t^z)$, diagnostic agents perceive the distribution of next-period productivity as

$$\log z_{t+1}^T \mid (\log z_t^T, \varepsilon_t^z) \sim N(\rho_z \log z_t^T + \theta \rho_z \varepsilon_t^z, \sigma_z^2).$$

Social Planner's problem We consider a social planner who directly controls the level of bond holdings, taking labor and goods market clearing conditions as given. The rational planner's problem can be written recursively as follows:

$$V^{sp}(z^T, b) = \max_{b', c^T c^N, h^T, h^N} u(c(c^T, c^N)) + \beta E\left[V^{sp}(z^{T\prime}, b') \mid z^T\right] \tag{A.63}$$

subject to

$$c^T + b' = z^T (h^T)^\alpha + (1+r)b, \tag{A.64}$$

$$c^N = z^N (h^N)^\alpha, \tag{A.65}$$

$$\bar{h} = h^T + h^N, \tag{A.66}$$

$$b' \geq -\kappa \left(z^T (h^T)^\alpha + \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N} \right)^{\eta+1} z^N (h^N)^\alpha \right), \tag{A.67}$$

$$\frac{z^T}{z^N} \left(\frac{h^T}{h^N} \right)^{\alpha-1} = \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N} \right)^{\eta+1}, \tag{A.68}$$

where equation (A.68) is an implementability constraint that links firms' optimal labor demand and households' optimal intratemporal consumption choices. In equilibrium, this condition ensures that the relative marginal product of labor across sectors equals the marginal rate of substitution between tradable and non-tradable consumption, as implied by competitive labor and goods markets.

To obtain the optimal debt tax, we identify the policy that decentralizes the rational social planner's allocations in an economy where households have DE.[20] Following the approach in Arce et al. (2025), in states where the borrowing constraint of the rational social planner is not binding, the optimal debt tax implemented by a paternalistic (rational) policymaker on the diagnostic households is given by

$$\tau_t^{sp} = \frac{E_t\left[u_T(t+1) + \mu_{t+1}^{sp} \widetilde{\Psi}_{t+1}\right]}{E_t^\theta [u_T(t+1)]} - 1. \tag{A.69}$$

where μ_t^{sp} denotes the Lagrange multiplier on the rational social planner's borrowing constraint

[20] The problem of the diagnostic social planner is identical to that of the rational social planner, except that expectations are formed diagnostically. As a result, the diagnostic planner's decisions depend not only on the current productivity level but also on the realized productivity innovation, ε^z, which becomes an additional state variable.

in (A.67), and $\widetilde{\Psi}_t$ is the collateral elasticity term, defined as

$$\widetilde{\Psi}_t = \underbrace{\kappa(\eta+1)\left(\frac{p_t^N c_t^N}{c_t^T}\right)}_{\equiv \psi_t} \underbrace{\frac{\frac{1-\alpha}{\alpha}\frac{\bar{h}}{h_t^T}}{\frac{1-\alpha}{\alpha}\frac{\bar{h}}{h_t^T} + (\eta+1)\left(\frac{p_t^N c_t^N}{c_t^T}+1\right)}}_{\equiv \Upsilon_t}. \tag{A.70}$$

The term ψ_t coincides with the pecuniary externality term in the baseline endowment economy, capturing how an increase in tradable consumption raises collateral values through higher non-tradable prices. The additional term Υ_t is specific to the production economy and reflects an endogenous labor reallocation mechanism: when tradable consumption increases, the relative price of non-tradable goods rises. Consequently, labor shifts toward the non-tradable sector, dampening the sensitivity of collateral values to borrowing. As a result, the effective collateral elasticity $\widetilde{\Psi}_t$ is attenuated relative to the endowment economy. Here, τ_t^{sp} can also be decomposed into two components as in Proposition 6:

$$\tau_t^{sp} = \frac{E_t[u_T(t+1)]}{E_t^\theta[u_T(t+1)]}\frac{E_t[\mu_{t+1}^{sp}\widetilde{\Psi}_{t+1}]}{E_t[u_T(t+1)]} + \frac{E_t[u_T(t+1)] - E_t^\theta[u_T(t+1)]}{E_t^\theta[u_T(t+1)]}, \tag{A.71}$$

$$= \underbrace{\frac{E_t[u_T(t+1)]}{E_t^\theta[u_T(t+1)]}\tau_t^{\star,sp}}_{\text{PE-DE interaction corrected}} + \underbrace{\frac{E_t[u_T(t+1)] - E_t^\theta[u_T(t+1)]}{E_t^\theta[u_T(t+1)]}}_{\text{DE corrected}}, \tag{A.72}$$

where $\tau_t^{\star,sp}$ denotes the benchmark debt tax implemented by the rational social planner on the rational households in the production economy.

Moreover, in states where the borrowing constraint of the diagnostic social planner is not binding, the optimal debt tax of a non-paternalistic (diagnostic) policymaker is

$$\tau^{\theta,sp} = \frac{E_t^\theta\left[\mu_{t+1}^{\theta,sp}\widetilde{\Psi}_{t+1}^\theta\right]}{E_t^\theta[u_T(t+1)]}, \tag{A.73}$$

where $\mu_t^{\theta,sp}$ denotes the Lagrange multiplier on the diagnostic social planner's borrowing constraint and $\widetilde{\Psi}_t^\theta$ is the collateral elasticity term perceived by the diagnostic planner.

We solve the model using the time iteration method under the baseline calibration of Arce et al. (2025) with the diagnosticity parameter $\theta = 1.415$ as in our baseline endowment economy.

Cyclicality of Optimal Debt Tax In the endowment economy, households perceive positive news regarding their endowments as a significant wealth effect, leading them to increase consumption and associated borrowing. Consequently, households overborrow excessively in re-

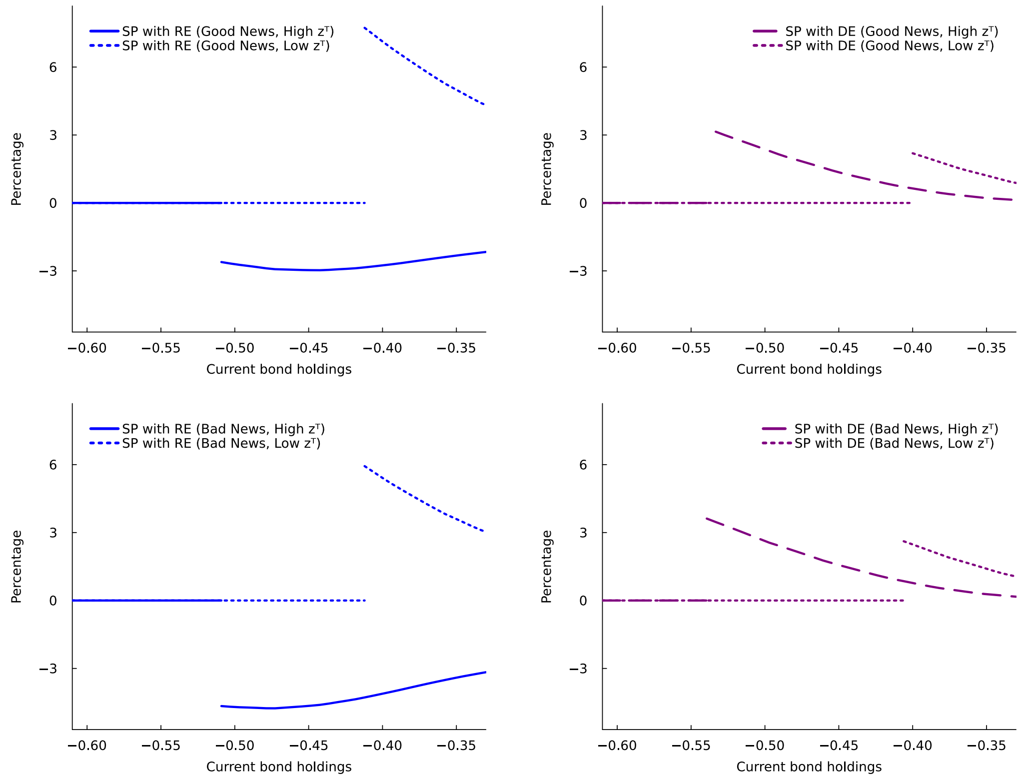

Figure A.1: Optimal Debt Tax Schedule

Notes: This figure shows the optimal tax schedule of the paternalistic planner (SP with RE) and the non-paternalistic planner (SP with DE) when the tradable productivity state is one standard deviation below and above the trend.

sponse to good news due to belief distortions, which necessitates a higher optimal debt tax. This expectation-correction channel is central to generating a countercyclical optimal debt tax.

However, in the production economy, the countercyclicality of optimal macroprudential policy is dampened. This is primarily because diagnostic households do not respond as strongly to the current realized news, thereby reducing the scope for macroprudential policy to address belief distortions. Since households receive income in the form of wages and profits—both of which they take as given—their incentives are tied to firm behavior. Households understand that firms will reallocate labor into the tradable sector in response to higher productivity in that sector, which in turn reduces revenue from the production of non-tradable goods. Thus, although households hold distorted beliefs regarding tradable sector productivity, they expect that profits distributed by firms will not increase significantly in response to good news. Additionally, labor reallocation toward the tradable sector reduces the price of non-tradable goods (p^N) for a given productivity level (z^T and z^N), as the relative marginal cost of producing non-tradable goods falls. This implies that labor reallocation tightens the borrowing constraint, preventing an excessive buildup of debt. Consequently, following positive news, households' per-

Table A.1: Cyclicality of Optimal Debt Tax: Corr(Output, Tax)

	Paternalism (SP with RE)	Non-Paternalism (SP with DE)
Baseline ($\theta = 1.415$)	-0.706	0.156
$\theta = 5.0$	-0.466	0.049
$\theta = 10.0$	-0.028	-0.071
$\theta = 15.0$	0.221	-0.127

Note: Correlations between output and the optimal debt tax for diagnostic households are reported for various values of the diagnosticity parameter. The simulated data come from the production economy of Arce et al. (2025), extended to incorporate diagnostic expectations.

ceived wealth and the collateral value do not increase as substantially as in the endowment economy, thereby weakening their incentive to overborrow.

Figure A.1 illustrates the optimal debt tax schedule in the production economy, confirming the weakened expectation-correction channel.[21] In the left panels, we present the paternalistic planner's optimal tax. The planner imposes a higher tax rate when the productivity level (z^T) is lower to address the pecuniary externality. Conversely, the planner raises the tax rate in response to positive news to mitigate overreactions stemming from belief distortions. Quantitatively, the tax variation driven by the pecuniary externality dominates that driven by belief distortions, substantially reducing the countercyclicality of the optimal debt tax. The right panels display the non-paternalistic planner's optimal tax, which, as expected, responds minimally to news. Interestingly, for a wide range of intermediate debt levels, the optimal tax is higher when productivity is high. This occurs because the borrowing constraint binds more easily under low productivity, driven by labor reallocation and the associated drop in non-tradable prices (p^N).

As the expectation-correction channel strengthens—that is, as the diagnosticity parameter θ increases—the countercyclicality of the paternalistic policymaker's optimal debt tax becomes more pronounced in the production economy as well. In particular, higher values of θ amplify the role of expectation-driven fluctuations relative to the pecuniary externality. When $\theta = 15.0$, the correlation between the optimal debt tax and output becomes positive, reaching 0.221, as reported in Table A.1. The requirement for such a high level of diagnosticity to generate a countercyclical tax arises because labor reallocation across sectors absorbs the direct pass-through of productivity shocks, thereby dampening wealth effects driven by overreactions to the realized news. Since such sectoral reallocation is unlikely to occur immediately in the presence of labor market frictions, the endowment economy provides useful guidance for the optimal implementation of macroprudential policy in the short run.

[21] Figures A.2 and A.3 present the corresponding decision rules for bond holdings of the social planner and the private agents under low and high productivity states, respectively.

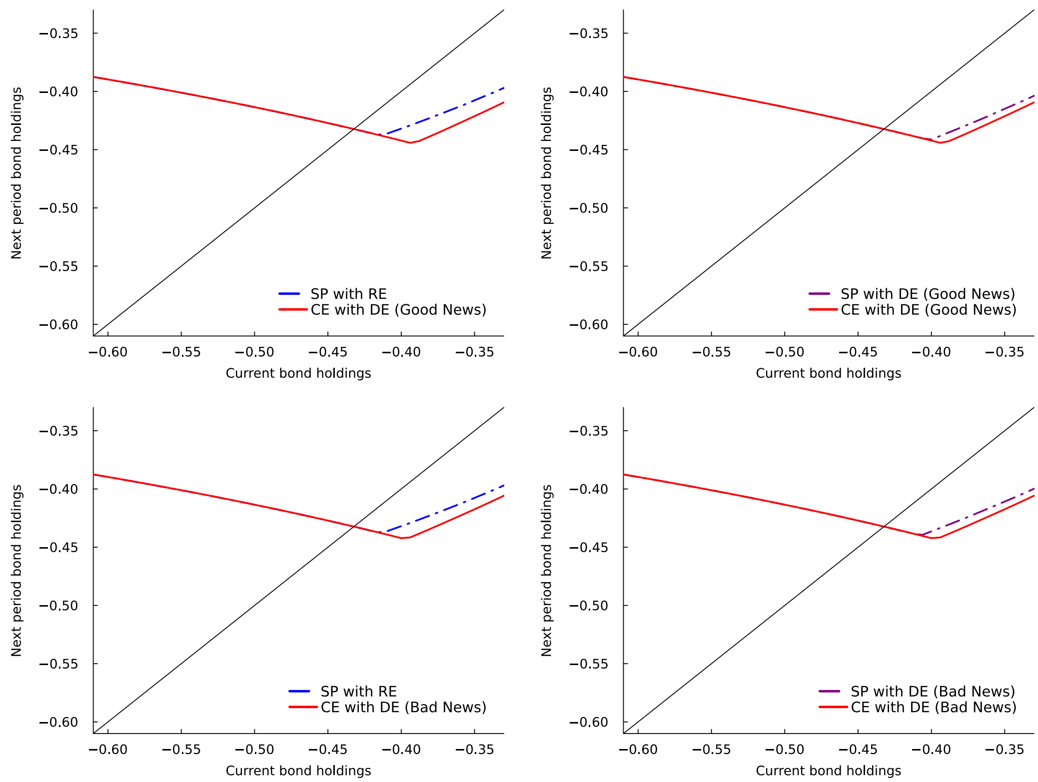

Figure A.2: Bond Decision Rules in a Low Productivity State

Notes: This figure shows the bond decision rules in the decentralized competitive equilibrium under the DE and in the constraint-efficient equilibrium when the tradable productivity state is one standard deviation below the trend.

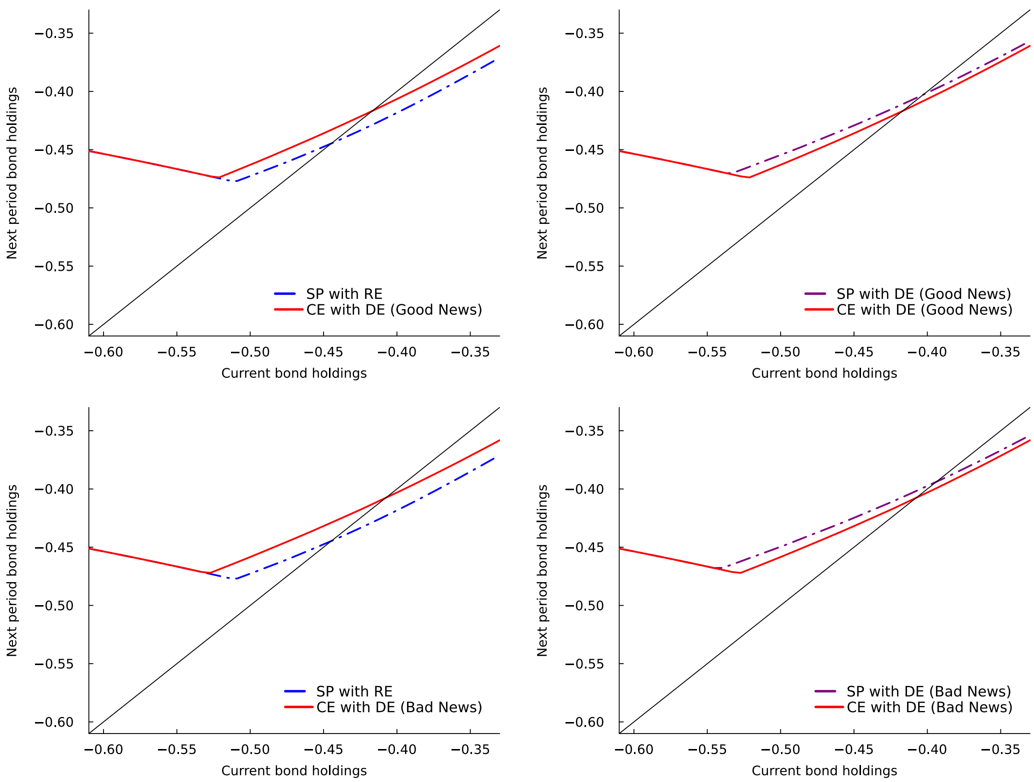

Figure A.3: Bond Decision Rules in a High Productivity State

Notes: This figure shows the bond decision rules in the decentralized competitive equilibrium under the DE and in the constraint-efficient equilibrium when the tradable productivity state is one standard deviation above the trend.

C.1 Rational Expectations Benchmark

Figure A.4 and Figure A.5 summarize the policy functions for labor allocation in the tradable sector, external bond holdings, and the optimal debt tax schedule under RE.

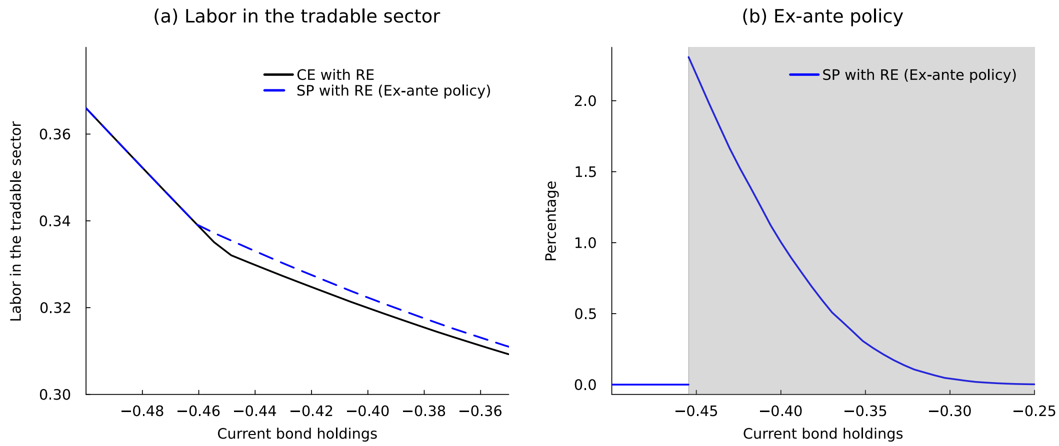

Figure A.4: Tradable Labor Policy function and Debt Tax Schedule

Notes: The panel (a) shows the tradable labor policy function in the decentralized competitive equilibrium and in the constraint-efficient equilibrium (ex-ante intervention) under the RE, and the panel (b) shows the optimal debt tax schedule implemented by the rational social planner on the rational agent, when the tradable productivity is set at the mean value.

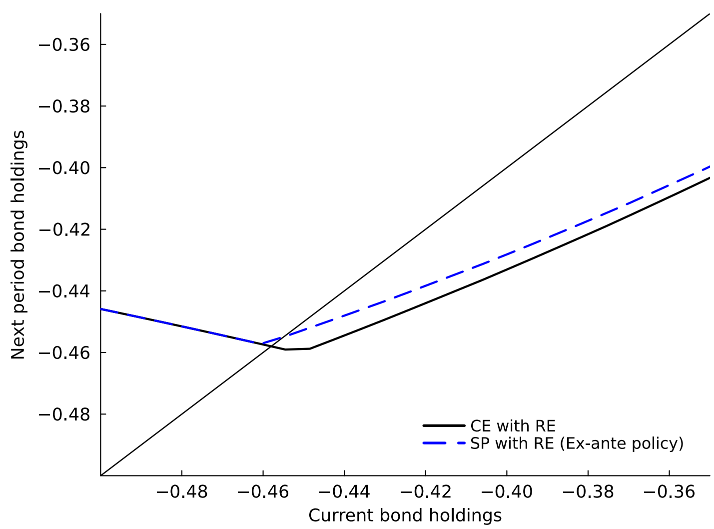

Figure A.5: Bond Decision Rules

Notes: This figure shows the bond policy function in the decentralized competitive equilibrium and in the constraint-efficient equilibrium (ex-ante intervention) under the RE, when the tradable productivity is set at the mean value.

D Solution Method

In this appendix, we describe the numerical procedures used to solve the competitive equilibrium and the social-planner problems. The competitive equilibrium is characterized by policy and pricing functions — $b'(y^T, \varepsilon^T, b)$, $p^N(y^T, \varepsilon^T, b)$, and $c^T(y^T, \varepsilon^T, b)$ — that satisfy:

$$u_T\big(c^T(y^T, \varepsilon^T, b), y^N\big) \geq \beta(1+r)E^{\theta}_{y^{T\prime}|y^T, \varepsilon^T} u_T\big(c^T\big(y^{T\prime}, \varepsilon^{T\prime}, b'(y^T, \varepsilon^T, b)\big), y^N\big), \tag{A.74}$$

$$p^N(y^T, \varepsilon^T, b) = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T(y^T, \varepsilon^T, b)}{y^N}\right)^{\eta+1}, \tag{A.75}$$

$$b'(y^T, \varepsilon^T, b) + \big(\kappa^N p^N\big(y^T, \varepsilon^T, b\big) y^N + \kappa^T y^T\big) \geq 0, \tag{A.76}$$

$$c^T(y^T, \varepsilon^T, b) = b(1+r) + y^T - b'(y^T, \varepsilon^T, b). \tag{A.77}$$

Discretization Before solving the model, we discretize the true AR(1) process for tradable income and the perceived diagnostic transition matrix of the exogenous states for each news state using the Tauchen method (Tauchen, 1986). The lower and upper bounds of the tradable income grid, $\bar{y}^T_1$ and $\bar{y}^T_{n_T}$, are set to m unconditional standard deviations around the unconditional mean μ^T:

$$\bar{y}^T_1 = \mu^T - m\sigma_T, \qquad \bar{y}^T_{n_T} = \mu^T + m\sigma_T.$$

Note that $m = 3$ in the general case and ε^T_t is Gaussian with mean zero and variance σ^2_ε.

Note that we do not choose the news grid arbitrarily. Instead, we recover the possible values of the news state from the discretized tradable endowment grid using $\bar{\varepsilon}^T_{ij} = \log \bar{y}^T_i - \rho^T \log \bar{y}^T_j$ for each $i, j \in \{1, 2, \ldots, n_T\}$, which yields up to n_T^2 implied news realizations. We then collapse this set to an n_T-point news grid by selecting nodes that preserve the original minimum and maximum. Let $\bar{\varepsilon}^T_1 < \cdots < \bar{\varepsilon}^T_{n_T}$ be the news grid. This procedure ensures that the news state space fully captures all realizable outcomes consistent with the endowment process.

Now, we define the diagnostic transition matrix as follows. Let $d \equiv \bar{y}^T_j - \bar{y}^T_{j-1}$ and $\Phi(\cdot)$ denote the standard normal cumulative distribution function. If j is between 2 and $n_T - 1$, the diagnostic transition probability matrix, $\pi^{\theta}_{ie,j} = \Pr\left[y^T_{t+1} = \bar{y}^T_j \mid y^T_t = \bar{y}^T_i, \varepsilon^T_t = \bar{\varepsilon}_e \right]$ is set by

$$
\begin{aligned}
\pi^{\theta}_{ie,j} &= \Pr\left[\bar{y}^T_j - d/2 \leq \mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e) + \varepsilon^T_{t+1} \leq \bar{y}^T_j + d/2 \right], \\
&= \Pr\left[\bar{y}^T_j - (\mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e)) - d/2 \leq \varepsilon^T_{t+1} \leq \bar{y}^T_j - (\mu_T + \rho_T(y^T_i + \theta\bar{\varepsilon}_e)) + d/2 \right], \\
&= \Phi\left(\frac{\bar{y}^T_j - (\mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e)) + d/2}{\sigma_\varepsilon} \right) - \Phi\left(\frac{\bar{y}^T_j - (\mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e)) - d/2}{\sigma_\varepsilon} \right).
\end{aligned}
$$

If $j = 1$, then

$$
\begin{aligned}
\pi^{\theta}_{ie,1} &= \Pr\left[\mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e) + \varepsilon^T_{t+1} \leq \bar{y}^T_1 + d/2 \right], \\
&= \Phi\left(\frac{\bar{y}^T_1 - (\mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e)) + d/2}{\sigma_\varepsilon} \right).
\end{aligned}
$$

If $j = n_T$, then

$$
\begin{aligned}
\pi^{\theta}_{ie,n_T} &= \Pr\left[\bar{y}^T_{n_T} - d/2 \leq \mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e) + \varepsilon^T_{t+1} \right], \\
&= 1 - \Phi\left(\frac{\bar{y}^T_{n_T} - (\mu_T + \rho_T(\bar{y}^T_i + \theta\bar{\varepsilon}_e)) - d/2}{\sigma_\varepsilon} \right).
\end{aligned}
$$

Time-Iteration Algorithm The competitive equilibrium is solved using a modified time iteration method to handle the asymmetric credit constraint.[22] We interpolate the functions using a piecewise liner approximation. Start the algorithm at an initial point by setting $K = 1$ and guess the equilibrium functions at this point, denoted by $c^T_K(y^T, \varepsilon^T, b)$, $p^N_K(y^T, \varepsilon^T, b)$ and $b'_K(y^T, \varepsilon^T, b)$. The algorithm is as follows:

1. Set the $b'_{K+1}(y^T, \varepsilon^T, b) = -(\kappa^N p^N_K(y^T, \varepsilon^T, b) y^N + \kappa^T y^T)$ as the maximum capacity of borrowing, and calculate $c^T_{K+1}(y^T, \varepsilon^T, b)$ using the market clearing condition for the tradable goods (A.77).

2. Compute the following equation:

$$
\begin{aligned}
EE &\equiv u_T\big(c^T_{K+1}(y^T, \varepsilon^T, b), y^N\big) - \beta(1+r) E^{\theta}_{y^{T\prime}|y^T,\varepsilon^T}\left[u_T\big(c^T_{K+1}\big(y^{T\prime}, \varepsilon^{T\prime}, b'_{K+1}(y^T, \varepsilon^T, b)\big), y^N\big) \right], \\
&= u_T\big(c^T_{K+1}(y^T, \varepsilon^T, b), y^N\big) \\
&\quad - \beta(1+r) \sum_{y^{T\prime}} \pi^{\theta}(y^{T\prime} \mid y^T, \varepsilon^T) u_T\big(c^T_{K+1}\big(y^{T\prime}, \underbrace{\varepsilon^{T\prime}}_{= \log y^{T\prime} - \rho^T \log y^T}, b'_{K+1}(y^T, \varepsilon^T, b)\big), y^N\big),
\end{aligned}
$$

[22] This method solves the model through backward recursion by iteratively substituting the model's optimality conditions expressed in a recursive way.

where $\pi^{\theta}(y^{T\prime} \mid y^T, \varepsilon^T)$ is the diagnostically distorted transition matrix. Note that although diagnostic agents form their expectations under the distorted distribution, they are aware that the true DGP for the tradable endowment follows an AR(1) process. This allows us to compute DE using the $\varepsilon^{T\prime} = \log y^{T\prime} - \rho^T \log y^T$.[23]

(a) If $EE > 0$, the credit constraint is binding; move to step (3).

(b) If $EE \leq 0$, the credit constraint is not binding. Then, solve for $c^T_{K+1}(y^T, \varepsilon^T, b)$ that satisfies the Euler equation (A.74) with equality using a root-finding algorithm. Then, using the market clearing conditions for the tradable goods, calculate the $b^{\prime}_{K+1}(y^T, \varepsilon^T, b)$.

3. Update $p^N_{K+1}(y^T, \varepsilon^T, b) = \frac{1-\omega}{\omega}\left(\frac{c^T_{K+1}(y^T, \varepsilon^T, b)}{y^N}\right)^{1+\eta}$.

4. Iterate these steps until the equilibrium functions, $b'(y^T, \varepsilon^T, b)$, $p^N(y^T, \varepsilon^T, b)$, and $c^T(y^T, \varepsilon^T, b)$, converge.

The algorithm of solving the social planner problem is similar to that of competitive equilibrium. The key difference is that u_T in step (2) becomes $u^{sp}_T = u_T + \mu^{sp}\psi$ where $\mu^{sp} > 0$ when the constraint binds and $\psi = \kappa^N(\eta+1)\left(\frac{1-\omega}{\omega}\right)\left(\frac{c^T}{y^N}\right)^{\eta}$ which arises from the internalization of the pecuniary externality.

[23] This differs from the ARMA(1,1) misperception case, where $\varepsilon^{T\prime}$ is given by $\varepsilon^{T\prime} = \log y^{T\prime} - \rho_T \log y^T - \theta\rho_T\varepsilon^T$.

E Effects of the Degree of Diagnosticity

To investigate the channels through which DE affect the model's characteristics, we explore how the degree of diagnosticity, θ, influences crisis dynamics.

Panel (b) of Figure A.6 illustrates the relationship between θ and the crisis probability, starting from values of θ close to zero ($\theta = 0.1$). Panel (a) of this figure also plots the ergodic distribution of bond holdings, revealing that the distribution becomes more dispersed as θ increases. This dispersion reflects the fact that households overborrow more frequently in response to good news, which increases the probability of a crisis. At the same time, because a financial crisis is defined as a period when the borrowing constraint binds, the crisis probability is measured as the cumulative density below the threshold bond-holding level. For values of θ smaller than the threshold (close to 1), the precautionary saving motive is sufficiently strong to lead the economy to hold larger bond positions more frequently, thereby reducing the crisis probability. Note that precautionary saving motives are further strengthened when the perceived variance of income increases in the presence of a borrowing constraint and/or convex marginal utility (Carroll et al., 2021). This highlights how the interaction between the borrowing constraint and DE generates nonlinear responses in the crisis probability relative to diagnosticity θ.

Figure A.7 shows that as the degree of diagnosticity increases, variations in aggregate variables including the depreciation of the exchange rate become larger. Intuitively, this is because a typical crisis becomes more severe as agents overreact more to the realized news. However, we find a nonlinearity for values of θ below 0.5. In this region, the precautionary saving motive dominates, leading to more pre-crisis savings. Consequently, the magnitude of the subsequent correction during a crisis becomes smaller.

Figure A.8 illustrates the average dynamics of the diagnostic competitive equilibrium around financial crises for different values of θ. As θ increases from 0.1 to 3.0, the economy experiences more severe booms in the periods leading up to a crisis. Standard models for studying Sudden Stop events do not easily generate such endogenous boom periods. Indeed, we match observed credit booms to calibrate the diagnosticity parameter θ. In our model, positive news shifts the probability density function of future tradable income ($y^{T\prime}$) rightward; this leads diagnostic agents to underestimate left-tail risks, thereby increasing the economy's vulnerability to crisis.

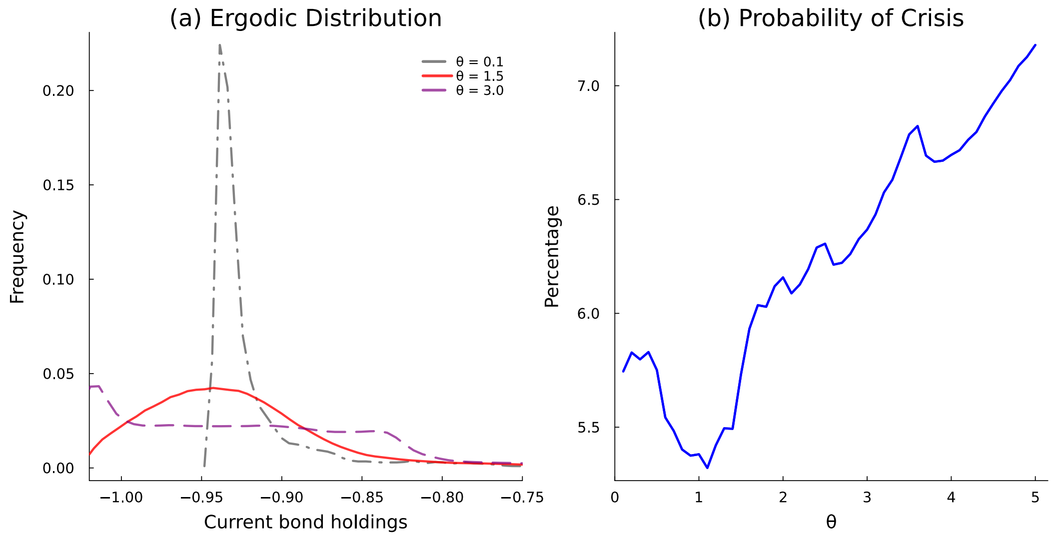

Figure A.6: Effects of the Degree of Diagnosticity (θ)

Notes: The ergodic distributions and the probability of crises are calculated by simulating one million stochastic time-series data, using the policy function from each value of θ.

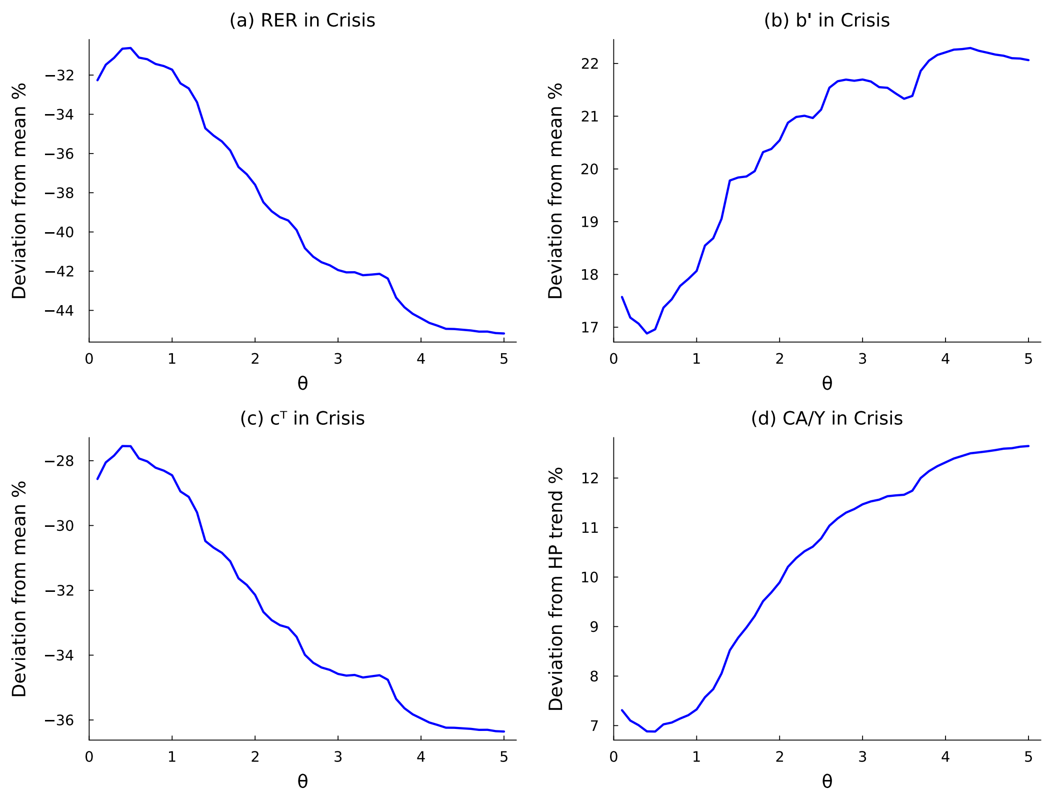

Figure A.7: Severity of Financial Crises and the Degree of Diagnosticity (θ)

Notes: This figure depicts the variation in financial crisis severity across different values of θ. The panels (a), (b), and (c) are plotted as percent differences relative to their ergodic mean. Panel (d) shows the current-account to GDP ratio in percentage. For each θ, the identification of Sudden Stops events follows the same methodology as in Figure 4.

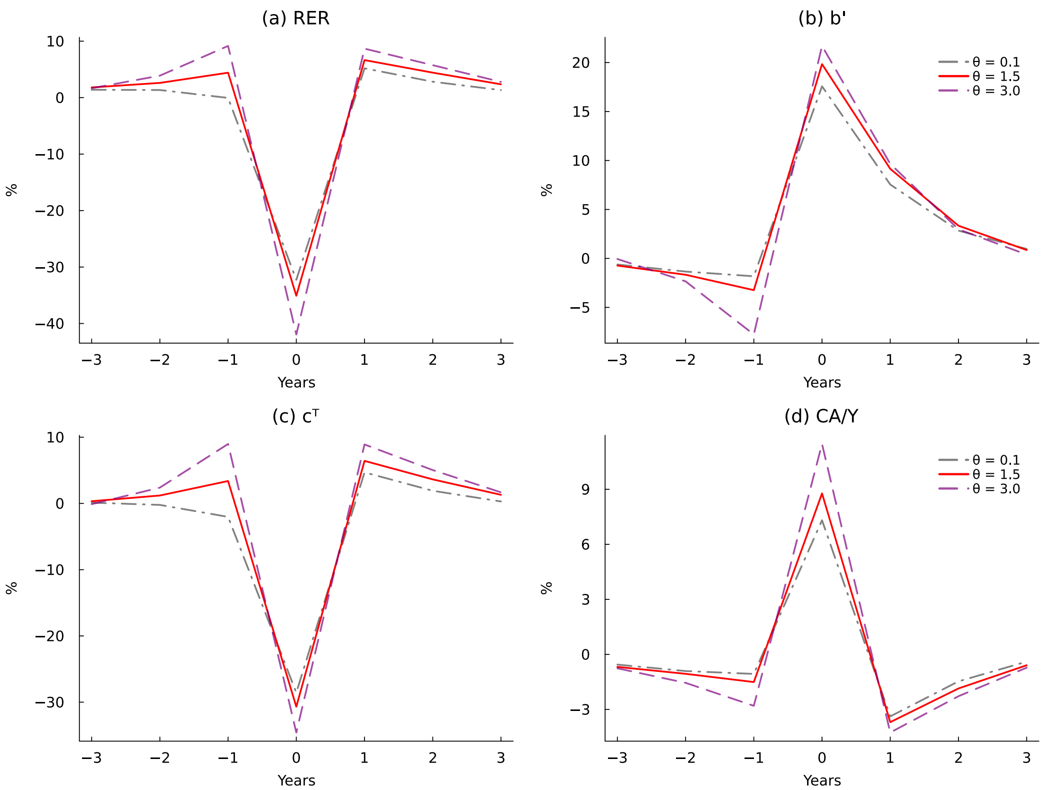

Figure A.8: Effects of the Degree of Diagnosticity (θ) on Crises Dynamics
Notes: This figure depicts the variation in financial crisis dynamics across different values of θ. For each θ, the identification of Sudden Stops events follows the same methodology as in Figure 4. The panels (a), (b), and (c) are plotted as percent differences relative to the ergodic mean. Panel (d) shows HP detrended current-account to GDP ratio with a smoothing parameter 10.

Figure A.9 presents the degree of overborrowing and crisis probabilities calculated for each solution under different parameterizations of the diagnosticity parameter (θ) and the parameter of the borrowing constraint ($\kappa = \kappa^T = \kappa^N$). The degree of overborrowing is measured as percentage deviations of the ergodic mean of leverage ($-b/y$) in the competitive equilibrium from that in the paternalistic social planner's allocation. We find interesting interactions between θ and κ and the associated nonlinearity. In panel (a), the overborrowing behaviors are intensified as the degree of diagnosticity increases, although there are some nonlinearities. Moreover, the degree of overborrowing increases at higher rates when the borrowing constraint parameter κ falls (the borrowing constraint becomes tighter). We observe similar patterns for crisis probabilities in panel (b), although nonlinearities become more pronounced due to a complex interaction between the threshold for tail risk and the distribution of debt as explained in the previous section. These results suggest that emerging market economies—who face lower κ than advanced economies possibly due to lower financial development—are more vulnerable to financial crises as agents' beliefs become more distorted.

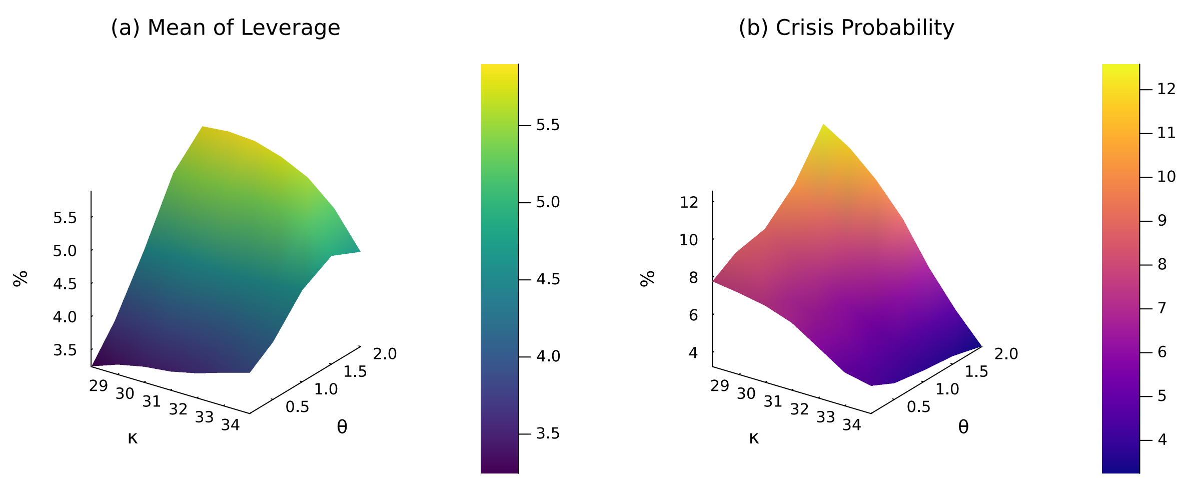

Figure A.9: Alternative Parameterization: Overborrowing and Crises Probabilities
Notes: In panel (a), for each parameterization of the diagnosticity parameter (θ) and the parameter of the borrowing constraint (κ), the degree of overborrowing is measured as percentage deviations of the ergodic mean of leverage ($-b/y$) in the competitive equilibrium from that in the paternalistic social planner's allocation.

Figure A.10 plots the severity of crises measured in terms of abrupt changes in key variables. The panels (a), (b), and (c) (real exchange rates, bond holdings, and tradable consumptions) are plotted as percent differences relative to their ergodic mean in absolute terms. Panel (d) shows the current-account to GDP ratio in percentage. The higher the values are, the more severe crises (more depreciations, more increase in bond holdings, further drops in tradable consumption, more sharp current account reversal). We generally find that crises are more severe when θ is higher and κ is higher, although nonlinearities occur. This implies that advanced economies with high financial development (κ) experience more severe crises once crisis events are realized, while crisis probabilities in advanced economies are lower than in emerging market economies. This pattern is more pronounced if households become more diagnostic.

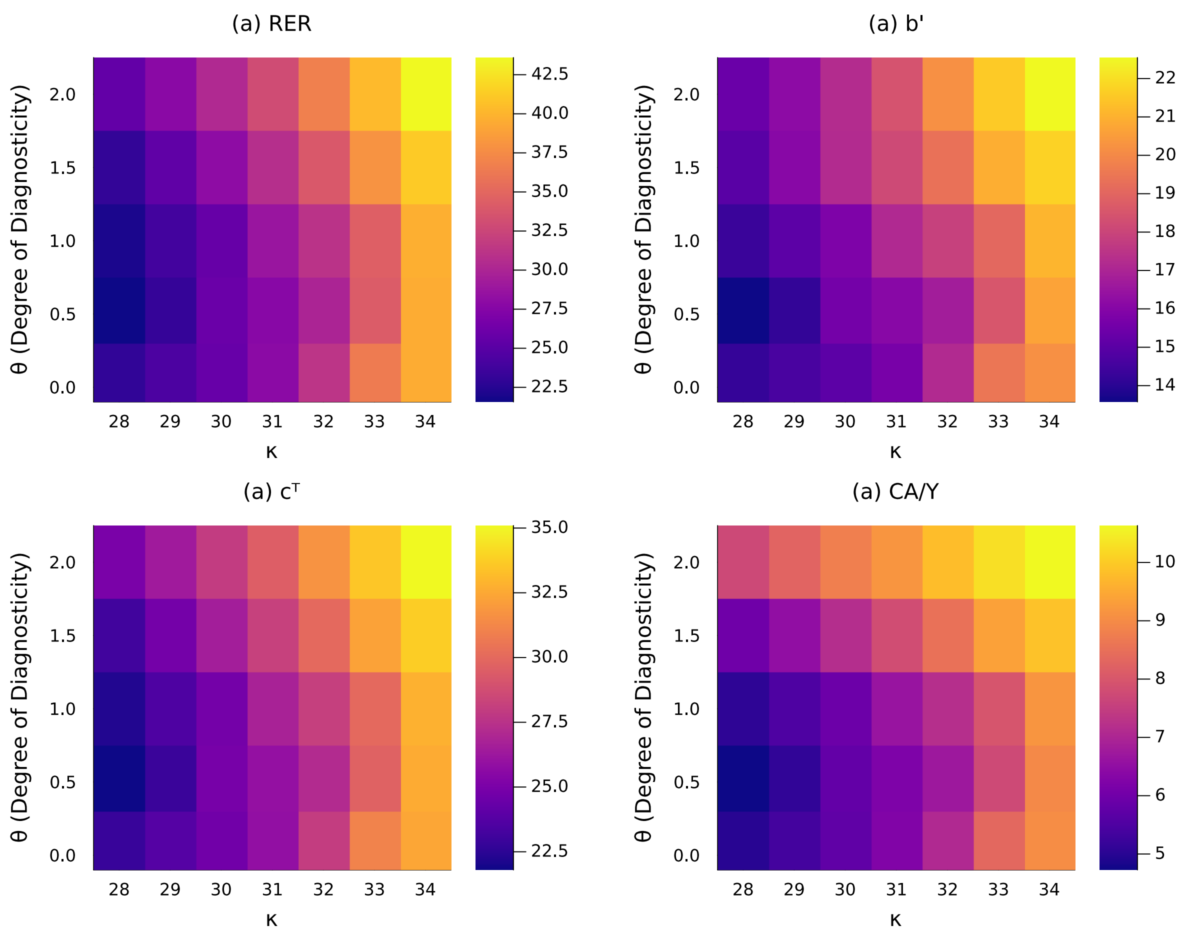

Figure A.10: Alternative Parameterization: Crises Dynamics

Notes: This figure depicts the variation in financial crisis severity across different sets of the diagnosticity parameter θ and the borrowing constraint parameter κ. The panels (a), (b), and (c) are plotted as percent differences relative to their ergodic mean in absolute terms. Panel (d) shows HP detrended current-account to GDP ratio with a smoothing parameter 10. In all panels, higher values imply more severe crises. For each θ and κ, the identification of Sudden Stops events follows the same methodology as in Figure 4.

F Sensitivity Analysis

This section investigates sensitivity of the model regarding alternative parameterizations, including the degree of diagnosticity θ, discount factor β, interest rate r, the borrowing constraint parameter κ, and persistence ρ^T and volatility σ^T of income process.

Table A.2 presents results on welfare gains, average debt tax rates, correlation between output and debt tax, correlation between debt and debt tax, and mean of leverage under various equilibria. Regarding welfare gains, we calculate the consumption needed to compensate the agent (in percentage change) so that she wants to move from the allocation set by the social planner to the allocation implied by the competitive equilibrium with diagnostic households. Such welfare gains are calculated for two cases: rational social planner (RE-SP) and diagnostic social planner (DE-SP) where the reference economy is in the competitive equilibrium with diagnostic households. For each type of social planner, we also obtain average debt tax rates, correlation between output and optimal debt tax, and correlation between debt and optimal debt tax, using simulated data from the model. The mean of leverage is calculated for each allocation under (i) competitive equilibrium with rational households (RE-CE), (ii) constrained Pareto optimum with the rational social planner (RE-SP), (iii) competitive equilibrium with diagnostic households (DE-CE), and (iv) constrained Pareto optimum with the diagnostic social planner (DE-SP).

Table A.3 shows similar results on the probability of crises along with severity of those crises measured in terms of changes in consumption, real exchange rate (RER), and current account to GDP ratio (CA/y).

Table A.2: Sensitivity Analysis: Tax Moments

	Welfare Gain (%)		Tax on Debt		Corr(Output,Tax)		Corr(Debt,Tax)		Debt-to-Output Ratio			
									RE		DE	
	RE SP	DE SP	RE SP	DE SP	RE SP	DE SP	RE SP	DE SP	CE	SP	CE	SP
baseline (θ=1.415)	0.15	0.08	2.07	4.12	0.53	-0.82	0.001	0.06	29.36	28.35	29.81	28.8
$\theta = 3$	0.38	0.02	-1.57	4.18	0.63	-0.41	-0.04	0.56	–	–	29.83	29.27
$\theta = 2$	0.25	0.06	0.74	4.24	0.60	-0.69	-0.02	0.30	–	–	29.90	29.11
$\theta = 1$	0.09	0.08	2.87	4.07	0.33	-0.84	0.05	-0.05	–	–	29.64	28.57
$\theta = 0.1$	0.08	0.09	4.09	4.16	-0.75	-0.77	0.09	0.05	–	–	29.38	28.37
$\beta = 0.93$	0.06	0.05	0.54	2.18	0.52	-0.85	0.10	-0.03	28.65	27.65	29.05	28.05
$\beta = 0.89$	0.22	0.11	3.86	5.97	0.55	-0.75	-0.04	0.13	29.75	28.73	30.12	29.2
$r = 0.06$	0.06	0.05	0.67	2.41	0.53	-0.85	0.10	-0.02	28.7	27.69	29.14	28.14
$r = 0.02$	0.19	0.11	3.72	5.7	0.54	-0.75	-0.04	0.13	29.75	28.73	30.07	29.17
$\kappa = 0.34$	0.13	0.09	2.14	4.32	0.53	-0.84	-0.002	0.06	30.46	29.34	30.81	29.8
$\kappa = 0.30$	0.14	0.06	2.17	3.85	0.52	-0.79	0.03	0.05	27.79	26.86	28.29	27.4
ρ_T (15 % more)	0.17	0.07	1.74	4.15	0.53	-0.76	-0.09	0.12	29.63	28.67	30.06	29.22
ρ_T (15 % less)	0.11	0.08	2.52	4.09	0.49	-0.84	0.15	0.02	29.16	28.07	29.5	28.4
σ_T (15 % more)	0.14	0.08	1.71	3.96	0.53	-0.83	0.04	0.02	28.78	27.63	29.3	28.16
σ_T (15 % less)	0.15	0.07	2.28	4.29	0.53	-0.79	-0.01	0.10	29.91	29.05	30.26	29.43

Table A.3: Sensitivity Analysis: Crises Moments

	Prob. Crisis				Severity of Financial Crises											
					Consumption				RER				CA/y			
	RE		DE		RE		DE		RE		DE		RE		DE	
	CE	SP	CE	SP	CE	SP	CE	SP	CE	SP	CE	SP	CE	SP	CE	SP
baseline (θ=1.415)	5.58	1.2	5.53	0.99	8.03	4.36	11.15	6.26	26.1	12.79	36.05	18.37	5.36	1.5	8.56	3.38
$\theta = 3$	–	–	6.5	2.91	–	–	12.8	9.38	–	–	42.59	28.5	–	–	11.3	7.55
$\theta = 2$	–	–	6.18	1.82	–	–	11.74	7.49	–	–	38.39	22.6	–	–	9.78	5.19
$\theta = 1$	–	–	5.4	0.77	–	–	10.24	5.67	–	–	32.81	16.5	–	–	7.27	2.47
$\theta = 0.1$	–	–	5.81	1.27	–	–	10.52	4.97	–	–	34	14.53	–	–	7.45	1.65
$\beta = 0.93$	4.16	0.87	2.34	0.36	10.4	5.8	10.34	6.13	34.62	17.35	32.94	17.8	6.84	1.99	6.65	2.33
$\beta = 0.89$	7.26	1.46	8.2	1.67	7.63	3.93	11.21	6.3	24.69	11.45	36.3	18.5	5.43	1.52	9.19	3.88
$r = 0.06$	4.47	0.85	2.63	0.44	10.14	5.71	10.75	6.27	33.58	17.05	34.38	18.2	6.54	1.92	7.06	2.48
$r = 0.02$	7.22	1.51	7.56	1.65	8.15	4.21	11.03	6.35	26.68	12.42	35.88	18.6	5.94	1.71	8.98	3.88
$\kappa = 0.34$	0.94	0.94	4.05	0.73	9.89	4.96	12.26	6.62	33.78	14.73	40.74	19.6	7.07	1.78	9.47	3.43
$\kappa = 0.30$	7.19	1.41	7.87	1.46	6.18	3.64	9.18	5.69	19.2	10.49	28.53	16.48	3.76	1.14	6.82	3.16
ρ^T (15% more)	6.3	1.2	6.04	1.31	6.84	3.89	10.83	6.57	22.01	11.32	34.78	19.4	4.5	1.47	8.7	4.15
ρ^T (15% less)	5.49	1.24	4.98	0.78	9.48	4.81	10.81	5.94	31.28	14.24	34.97	17.3	6.43	1.53	7.75	2.63
σ^T (15% more)	5.24	1.22	4.35	0.74	9.78	5.48	12.15	7.11	33.14	16.49	40.09	21	6.51	1.9	8.9	3.51
σ^T (15% less)	6.91	1.31	6.92	1.32	6.75	3.53	9.89	5.42	21.36	10.21	31.3	15.7	4.66	1.32	7.89	3.17

G Additional Results

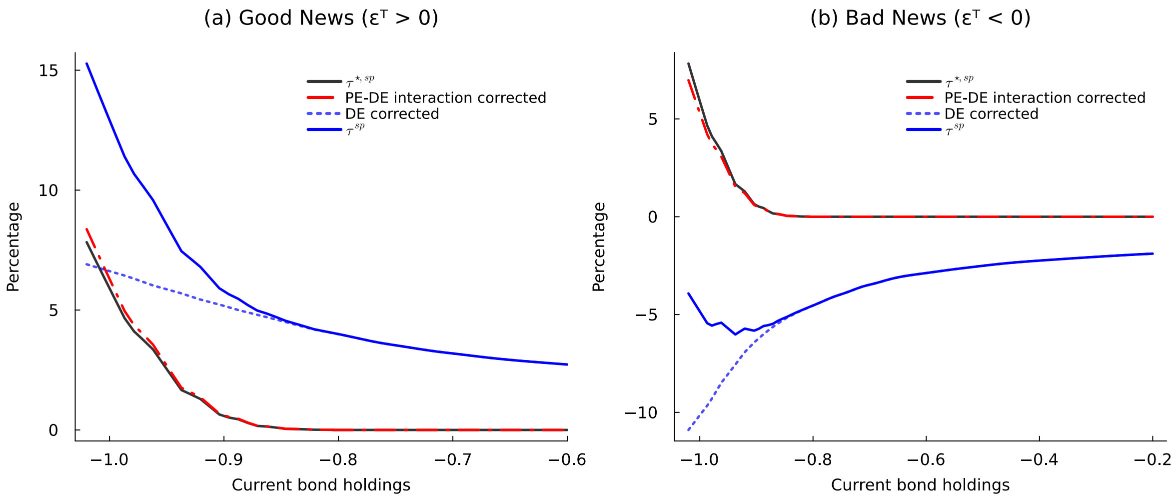

Figure A.11: Optimal Tax Decomposition in a High Income State
Notes: This figure shows the decomposed optimal debt tax imposed by the paternalistic (rational) social planner on the diagnostic private agent, when the current tradable income state is one standard deviation above the trend. $\tau^{\star,sp}$ (black solid line) represents the benchmark optimal debt tax for the rational social planner with rational households as we define in equation (47). τ^{sp} (blue solid line) represents the optimal debt tax of the paternalistic policymaker.

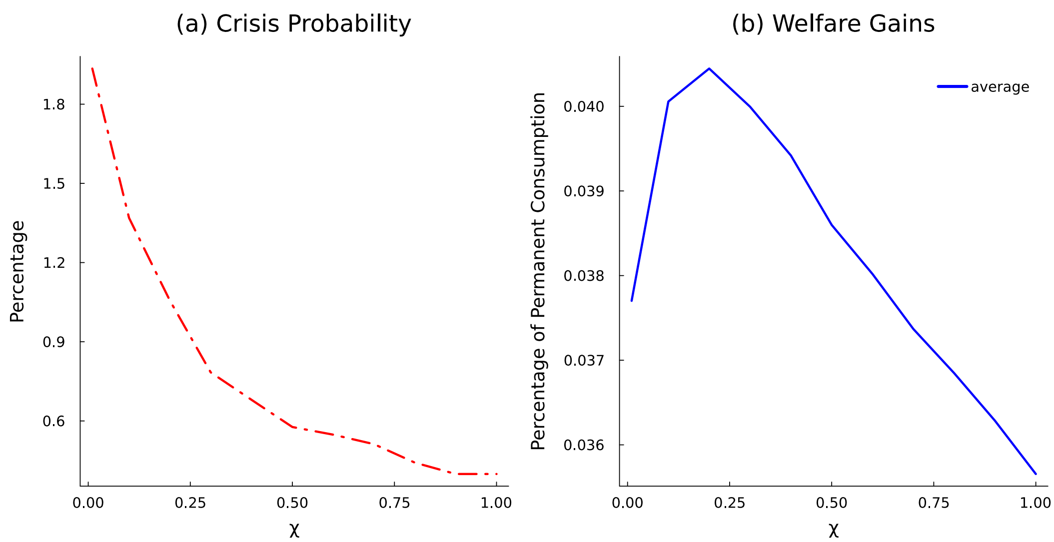

Figure A.12: Probability of Crises and Welfare Gains from Simple Debt Tax Policies
Notes: This figure shows the probability of crisis and the welfare gains as a function of χ, which is the elasticity of the tax with respect to the excess of the borrowing compared to a target $\bar{b} = -0.84$. Welfare gains are computed as in equation (49) by replacing the value function of social planner with the regulated competitive equilibrium, adjusted by the simple tax rule.

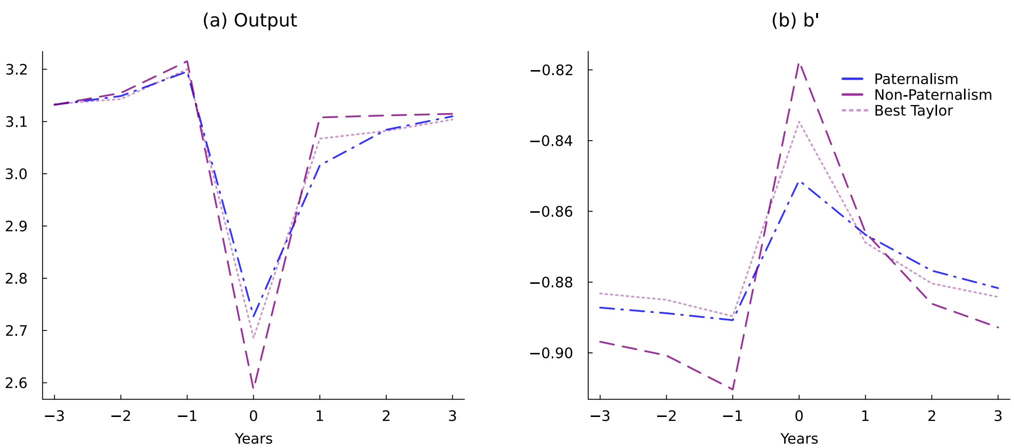

Figure A.13: Crisis Dynamics in Levels: Optimal vs. Simple Policies

Notes: This figure displays the average dynamics of various tax policies surrounding Sudden Stop events, presented in levels. A financial crisis is defined as a period in which the credit constraint binds and net capital outflows exceed one standard deviation of their ergodic mean in the competitive equilibrium under DE.

H Rational Expectations Benchmark

For comparison with the baseline model under DE, we also solve for the RE equilibrium and the corresponding social planner allocation. A further distinction from Bianchi (2011) is that the only source of uncertainty in our model arises from tradable income. However, this assumption does not alter the qualitative dynamics relative to Bianchi (2011), as illustrated in Figure A.14. The figure shows that the bond policy functions under a VAR(1) process and an AR(1) process with fixed y^N are qualitatively similar. Quantitatively, the VAR(1) specification implies less borrowing than the AR(1) process. The reason is that under the VAR(1) process, agents face an additional stochastic income source: even when tradable income is low, there remains a positive probability that non-tradable income will be high, which would reduces the incentive to borrow. Figure A.15 and A.16 plot bond decision rules and optimal debt taxes along with associated welfare gains under the rational competitive equilibrium and the rational social planner.

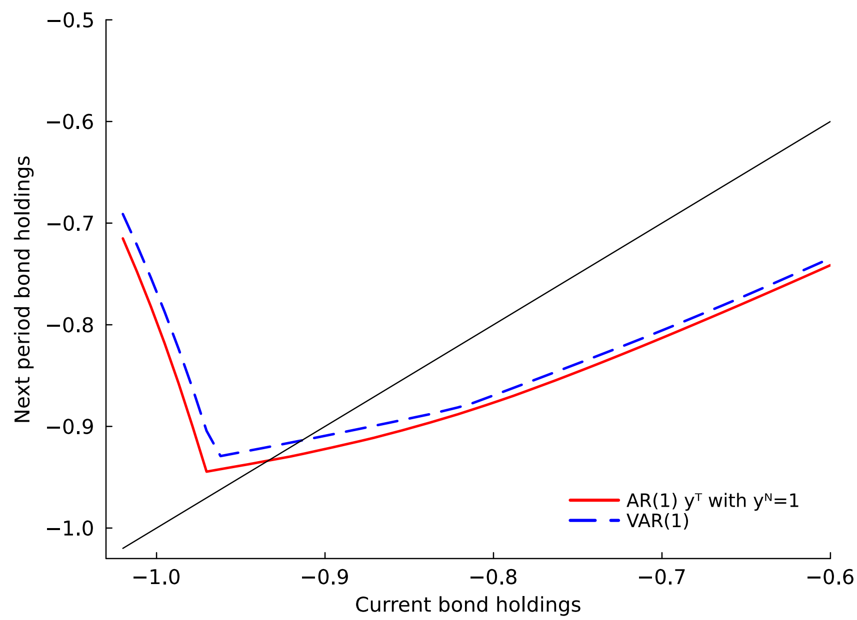

Figure A.14: Bond Decision Rules: VAR(1) process vs. AR(1) process with a fixed y^N

Notes: This figure compares the bond decision rules in the competitive equilibrium under RE using both an AR(1) process with $y^N = 1$ and a VAR(1) process as in Bianchi (2011) when the current tradable income state is one standard deviation below the trend and the non-tradable income state is set at the its mean value.

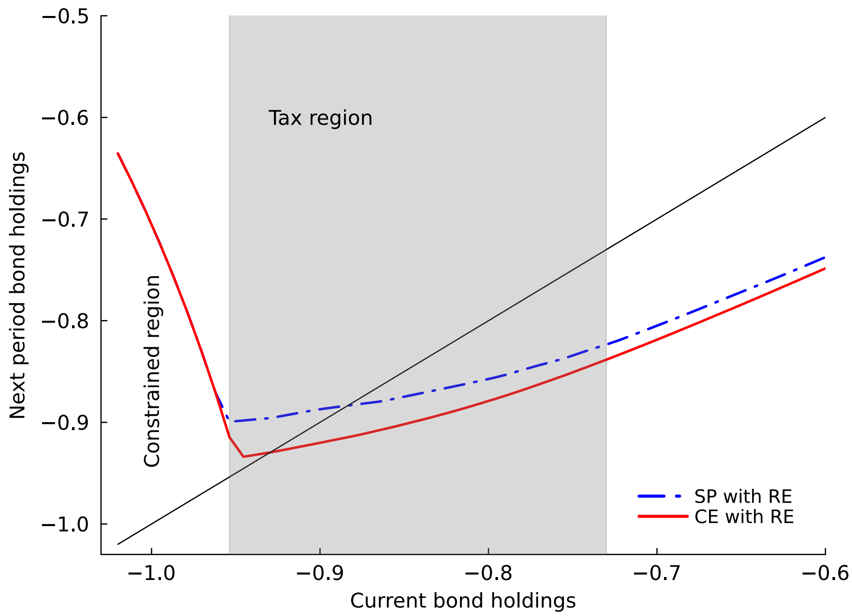

Figure A.15: Bond Decision Rules in a Low Income State
Notes: This figure shows the bond decision rules in the decentralized competitive equilibrium under the RE and in the constraint-efficient equilibrium under the RE when the current tradable income state is one standard deviation below the trend.

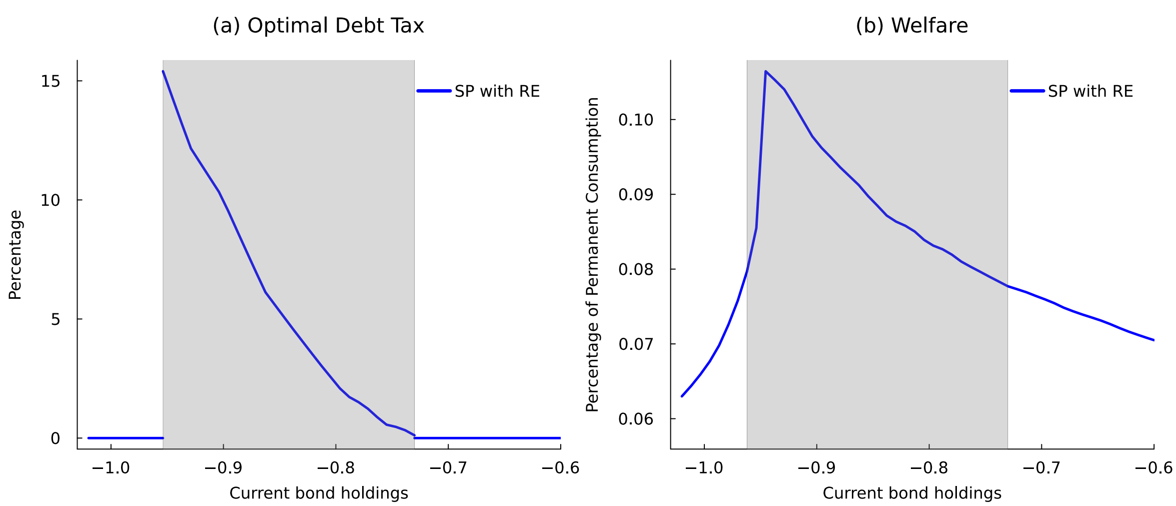

Figure A.16: Tax Schedule and Welfare Gains in a Bad Income State
Notes: This figure shows the welfare gains from correcting the externality and the optimal tax schedule implemented by the rational planner on the rational households, when the current tradable income state is one standard deviation below the trend. The grey-shaded region indicates the optimal tax is positive. Welfare gains are computed as compensating consumption variations that equalize expected utility across the competitive equilibrium and the constraint-efficient equilibrium under the RE in both equilibria.